<u>WORK AND ENERGY</u>

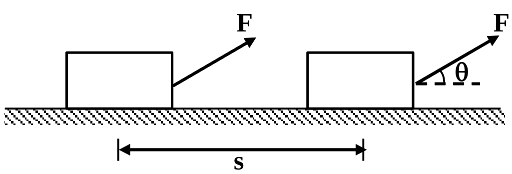
<u>JAMES PRESCOTT JOULE :</u>

James Prescott Joule, (1818 –1889) was an English physicist, who studied the nature of heat, and discovered its relationship to mechanical work. This led to the theory of conservation of energy, which led to the development of the first law of thermodynamics. The SI unit of work, the joule, is named after him. He worked with Lord Kelvin to develop the absolute scale of temperature, made observations on magnetostriction, and found the relationship between the flow of current through a resistance and the heat dissipated, now called Joule's law.

5.1 Work

The work W done by a constant force F when its *point of application* undergoes a displacement s is defined to be

$$W = F\, s \cos\theta$$



The work W done by the force $\boldsymbol{F}$ when its point of application undergoes a displacement $\boldsymbol{s}$ is

$W = \vec{\mathbf{F}}.\vec{\mathbf{s}} = Fs \cos\theta$

where θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ as indicated in figure. Only the component of $\vec{\mathbf{F}}$ along s, that is , $F \cos\theta$, contributes to the work done. Strictly speaking, the work is done by the *source* or *agent* that applies the force. Work is a *scalar* quantity and its *SI* unit is the *joule (J)*.

Work is also defined as the *dot product of force and its displacement* as given by equation

$$W = \vec{\mathbf{F}}.\vec{\mathbf{s}}$$

In terms of rectangular components, the two vectors are $\vec{\mathbf{F}} = F_x\hat{\boldsymbol{i}} + F_y\,\hat{\boldsymbol{j}} + F_z\,\hat{\boldsymbol{k}}$ and

$$\vec{\mathbf{s}} = \Delta x\,\hat{\boldsymbol{i}} + \Delta y\,\hat{\boldsymbol{j}} + \Delta z\,\hat{\boldsymbol{k}}\,;$$

hence, the definition of work may be written as

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

The work done by a given force on a body depends only on the *force*, the *displacement*, and the *angle* between them. It does not depend on the velocity or the acceleration of the body, or on the presence of other forces.

Since the work is a scalar, its value also does not depend on the orientation of the coordinate axes. Since the magnitude of a displacement in a given time interval depends on the velocity of the *frame of reference* used to measure the displacement, *the calculated work also depends on the reference frame*.

ILLUSTRATIONS

***Illustration* 1**

A box is moved over a horizontal path by applying force F = 60 N at an angle □ = 30° to the horizontal. What is the work done during the displacement of the box over a distance of 0.5 km.

Solution

By definition, $W = F\,s\cos$ □

Here F = 60 N; s = 0.5 km = 500 m; □ = 30°.

□ $W = (60)(500)\cos 30^o = 26$ kJ

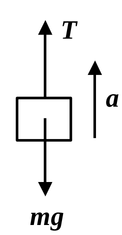
***Illustration* 2**

A load of mass m = 3000 kg is lifted by a winch with an acceleration a = 2 m/s^2. Find the work done during the first 15s from the beginning of motion.

Solution

The height to which the body is lifted during the first t second is h = $\frac{1}{2}\text{at}^2$.

The tension in the rope is given by



$$T = mg + ma$$

Work done is given by

$$W = Th = m(g + a)\left(\frac{1}{2}at^2\right)$$

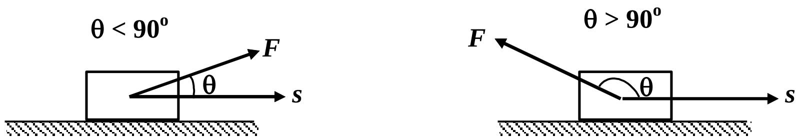
Here m = 3000kg, a = 2m/s^2; g = 10m/s^2; t = 1.5 s.

□ $W = (3000)(10 + 2)\left[\frac{1}{2}(2)(1.5)^2\right]$

or W = 81 kJ

Positive and Negative Work

Work done by a force may be positive or negative depending on the angle θ between the force and displacement. If the angle θ is acute ($\theta < 90^o$), then the work done is positive and the component of force is parallel to the displacement.



(a) Positive Work done by a force *F* **(b) Negative Work done by a force *F***

If the angle θ is obtuse ($\theta > 90^o$), the component of force is antiparallel to the displacement and the work done by force is negative.

Zero Work Done

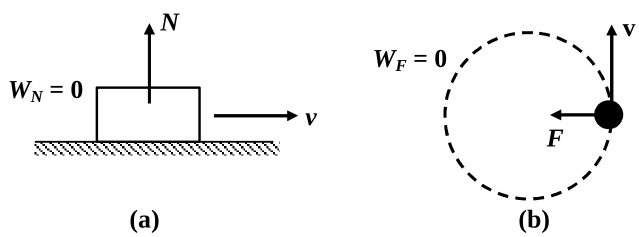
It is clear from the equation that the work done by a force is zero when

(a) $F = 0$

(b) $s = 0$

(c) $\cos\theta = 0$

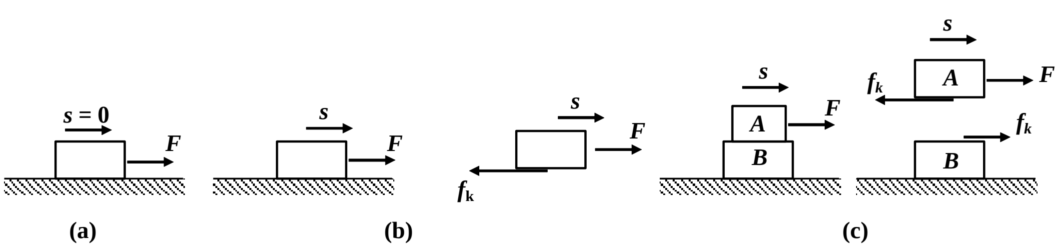
or $\theta = 90^o$



When the force and the displacement are perpendicular, the work done by the force is zero.
(a) the normal reaction N is perpendicular to displacement, therefore, $W_N = 0$
(b) the centripetal force is perpendicular to displacement, thus $W_F = 0$.

Work Done by Friction

There is a misconception that the force of friction always does negative work. In reality, the work done by friction may be *zero*, *positive* or *negative* depending upon the situation as shown in the figure.



(a) When a block is pulled by a force F and the block does not move, the work done by friction is zero.

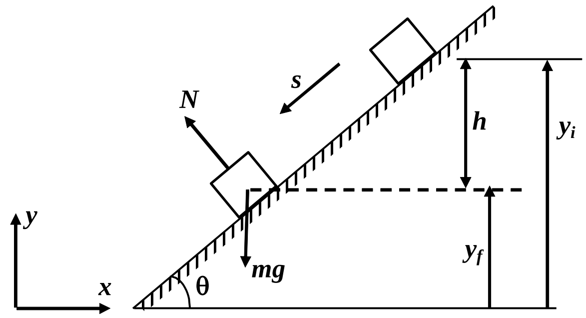
(b) When a block is pulled by a force F on a stationary surface, the work done by the kinetic friction is negative.

(c) Block A is placed on the block B. When the block A is pulled with a force F, the friction force does negative work on block A and positive work on block B, which is being accelerated by a force F. The displacement of A relative to the table is in the forward direction. The work done by kinetic friction on block B is positive.

Work Done by Gravity

Consider a block of mass m which slides down a smooth inclined plane of angle θ as shown in figure.

Let us assume the coordinate axes as shown in the figure, to specify the components of the two vectors - although the value of work will not depend on the orientation of the axes.



The work done by gravity is $W_g = -mg(y_f - y_i) = +mgh$

Now, the force of gravity, $\vec{\mathbf{F}}_g = -mg\,\hat{j}$

and the displacement is given by

$$\vec{\mathbf{s}} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

The work done by gravity is

$$W_g = \vec{\mathbf{F}}_g . \vec{\mathbf{s}} = -mg\,\hat{j} .(\Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k})$$

or $\quad W_g = -mg\Delta y \qquad (\because \hat{j} . \hat{i} = 0,\ \hat{j} . \hat{j} = 1,\ \hat{j} . \hat{k} = 0)$

Since $\Delta y = y_f - y_i = -h$

$\therefore \quad W_g = -mg(y_f - y_i) = +mgh$

If the block moves in the upward direction, then the work done by gravity is *negative* and is given by

$$W_g = -mgh$$

Important

1. The work done by the force of gravity depends only on the *initial* and *final* vertical coordinates, *not* on the path taken.
2. The work done by gravity is *zero* for any path that returns to its initial point.

When several forces act on a body one may calculate the work done by each force individually. *The net work done on the body is the algebraic sum of individual contributions.*

$$W_{net} = \vec{\mathbf{F}}_1 . \vec{\mathbf{s}}_1 + \vec{\mathbf{F}}_2 . \vec{\mathbf{s}}_2 + \ldots\ldots\ldots + \vec{\mathbf{F}}_n . \vec{\mathbf{s}}_n$$

or $\quad W_{net} = W_1 + W_2 + \ldots\ldots\ldots\ldots\ldots + W_n$

ILLUSTRATIONS

***Illustration* 3**

A ball of mass *m* is thrown as a projectile at an angle θ with the ground with velocity *u*. Determine the work done by gravity during the upward motion and also during the downward motion.

Solution

The maximum height of the projectile is $H = \frac{u^2 \sin^2 \theta}{2g}$

The work done by gravity during the upward motion is

$$w_g = -mgH = -\frac{mu^2 \sin^2 \theta}{2}$$

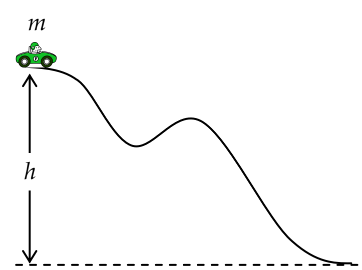
and work done during the downward motion is

$$w_g + mgH = \frac{mu^2 \sin^2 \theta}{2}$$

Note that the overall work done by gravity is zero because the net displacement of the ball in the vertical direction is.

***Illustration* 4**

A trolley-car starts from rest at the top of a hill as shown in figure and moves down the curved track. Determine the work done by gravity when it reaches the bottom.



Solution

As the trolley-car moves along the curved track, the only force acting on it is the force of gravity in downward direction. Since the work done by gravity is independent of path, therefore, the work done by gravity is the force multiplied by the displacement of the trolley-car in downward direction which is the height of the hill.

Hence $W_g = mgh$.

PRACTICE EXERCISE

1. A horizontal force of 25N pulls a box along a table. How much work does it do in pulling the box 80 cm?

2. A child pushes a toy box 4.0m along the floor by means of a force of 6N directed downward at an angle of 37° to the horizontal.

(a) What is the work done by the child?

(b) While displacing the box if the child pulls upward the box at the same angle then the work done by the child would be less or more?

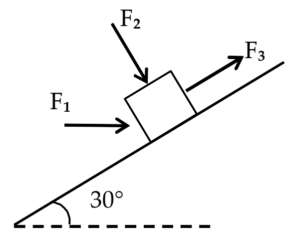
3. A block of mass 2.5kg moves upward on a smooth inclined plane by 80 cm. Three of the forces acting on the block are shown as F_1, F_2 and F_3.

F_1 = 40 N (horizontal)

F_2 = 20N (normal to plane)

F_3 = 30 N (parallel to plane)

Find the net work done by all the forces on the block.



ANSWERS

1. 20 J **2.** (a) 19.2 J (b) Less work **3.** 42 J

5.2 Work Done by a Variable Force

When the *magnitude* and *direction* of a force vary in three dimensions, it can be expressed as a function of the position vector $\vec{\mathbf{F}}(r)$, or in terms of the coordinates $\vec{\mathbf{F}}(x, y, z)$. The work done by such a force in an *infinitesimal* displacement *ds* is

$$W = \vec{\mathbf{F}}.d\vec{\mathbf{s}}$$

The total work done in going from point *A* to point *B* as shown in the figure. i.e.

$$W_{A\to B} = \int_A^B \vec{\mathbf{F}}.d\vec{\mathbf{s}} = \int_A^B (F\cos\theta)\, ds$$

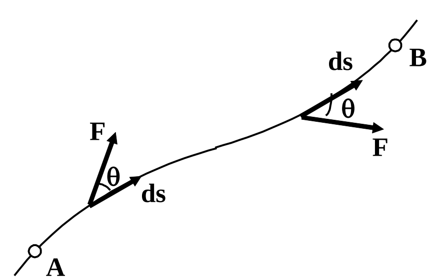
In terms of rectangular components,

$$\vec{\mathbf{F}} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

and $d\vec{\mathbf{s}} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

therefore ,

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$



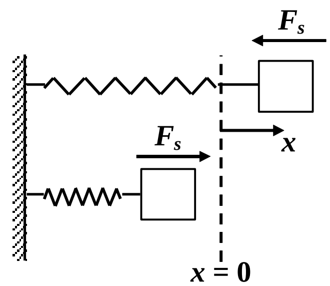
A particle moves along a curved path subject to a variable force $\vec{\mathbf{F}}$. The work done by the force in a displacement $d\vec{\mathbf{s}}$ is $dW = \boldsymbol{F}.d\boldsymbol{s}$.

Work done by a Spring

If x be the displacement of the free end of the spring from its equilibrium position then, the magnitude of spring force is given by

$$F_x = -kx$$

The *negative* sign signifies that the force always opposes the extension ($x > 0$) or the compression ($x < 0$) of the spring. In other words, the force tends to restore the system to its equilibrium position



The force exerted by an ideal spring is given by Hooke's law: $F_S = -kx$, where x is the *extension* or *compression* of the spring.

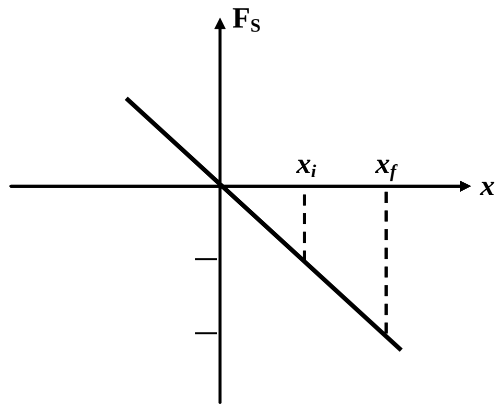
The work done by the spring force for a displacement from x_i to x_f is given by

$$W_s = \int_{x_i}^{x_f} F_s dx = -\int_{x_i}^{x_f} kx\, dx$$

or $$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Note

- The work done by a spring force is negative
- The work done by the spring force only depends on the *initial* and *final* points.
- The net work done by the spring force is *zero* for any path that returns to the initial point.

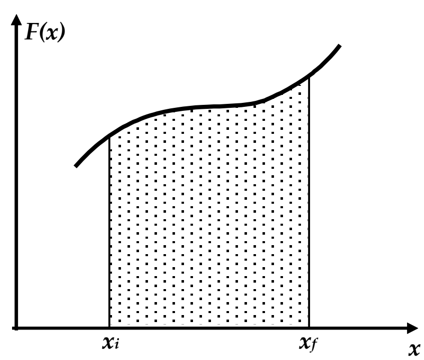


The work done by the spring when the displacement of its free end changes from x_i to x_f is the area of the trapezoid;

$$\mathbf{W_S} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Graphically, the work done by the spring force in a displacement from x_i to x_f is the *shaded area* (as shown in the figure 5.8) which is the difference in the areas of two triangles.

In *general*, the work done by a variable force *F(x)* from an initial point x_i to final point x_f is given by the *area under the force - displacement curve* as shown in the figure.



(a) The work done by a nonconstant force is approximately equal to sum of the areas of the rectangles.
(b) The area under the curve is given by the integral $W = \int F_x\, dx$

Area (work) above the x - axis is taken as *positive*, and vice-versa.

ILLUSTRATIONS

***Illustration* 5**

A force varying with distance is given as $F = ae^{-bx}$ acts on a particle moving in a straight line. Find the work done on the particle in its displacement from origin to a distance d.

Solution

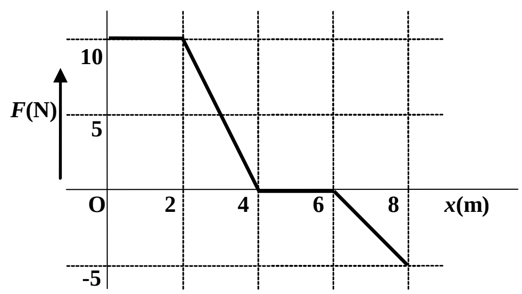
As the applied force varies with displacement, its work is given as $W = \int_0^d F \,.\, dx$

or $\quad W = \int_0^d ae^{-bx}\, dx$

$$W = -\frac{a}{b}\left[e^{-bx}\right]_0^d - \frac{a}{b}[-e^{-bd} - 1]$$

***Illustration* 6**

A 5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. Find the work done by this force as the block moves from the origin to x = 8m.



Solution

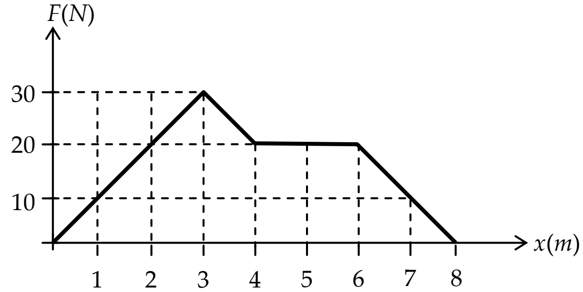
The work from x = 0 to x = 8 m is the area under the curve.

$$W = (10 \times 2) + \frac{1}{2}(10)(4-2) + 0 + \frac{1}{2}(\square 5)(8-6) = 25 \text{ J}$$

PRACTICE EXERCISE

4. A body is thrown on a rough surface such that friction force acting on it is linearly varying with distance traveled by it as $f = ax + b$. Find the work done by the friction on the box if the box travels a distance s, before coming to rest.

5. A body of 4 kg mass placed on a smooth horizontal surface experiences a force varying with displacement of block as shown in figure. Find the work done by the force upto x = 8m.



Answers

4. $\frac{1}{2}as^2 + bs$ **5.** 130 J

5.3 Work Energy Theorem

Let us study *what physical quantity changes when work is done on a particle*. If a constant force *F* acts through a displacement *x*, it does work $W = Fx = (ma)\,x$ on the particle.

Since the acceleration is constant, we can use the equation of kinematics

$$v_f^2 = v_i^2 + 2ax$$

Thus, $W = \dfrac{m\left[v_f^2 - v_i^2\right]}{2} = \dfrac{1}{2}mv_f^2 - \dfrac{1}{2}mv_i^2$

The quantity $K = \dfrac{1}{2}mv^2$ is a scalar and is called the *kinetic energy* of the particle.

Kinetic energy is the energy that a particle posses by virtue of its motion.

Thus, the equation takes the form

$$W = K_f - K_i = \Delta K$$

The work done by a force changes the kinetic energy of the particle. This is called the *Work-Energy Theorem.*

In *general*

The *net work done* by the resultant of all the force acting on the particle is equal to the change in *kinetic energy* of a particle.

W_{net} □□□□□□□

Important

(i) The kinetic energy of an object is a measure of the amount of work needed to increase its speed from zero to a given value.

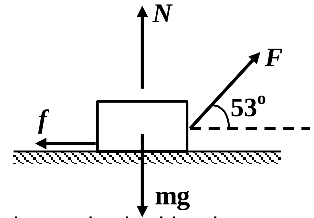
(ii) The kinetic energy of a particle is the work it can do on its surroundings in coming to rest.

(iii) Since the *velocity* and *displacement* of a particle depend on the *frame of reference*, the numerical values of the work and the kinetic energy also depend on the frame.

ILLUSTRATIONS

***Illustration* 7**

A block of mass m = 4 kg is dragged 2m along a horizontal surface by a force F = 30 N acting at 53° to the horizontal. The initial speed is 3 m/s and □$_k$ = 1/8.



The change in the kinetic energy of the block is given by the net work done on it.

(a) Find the change in kinetic energy of the block

(b) Find its final speed

Solution

(a) The forces acting on the block are shown in the figure. Clearly, $W_N = 0$ and $W_g = 0$, whereas

$W_F = Fs$ cos□ □

$W_f = -fs = -$□$_k Ns$ where $N = mg - F$sin□

The work-energy theorem,

□$K = W_{net} = W_F + W_f$

therefore,

□$K = F\,s$ cos□ - □$_k(mg - F$ sin□$)s$

$= (30)(2)(0.6) - \frac{1}{8}(40 - 24)(2) = 32$ J

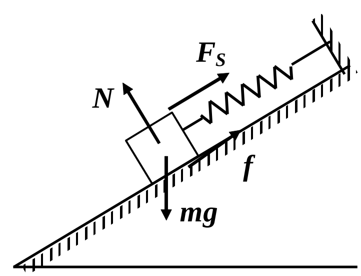
(b) Now □$K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 32$ J

Since v_i = 3 m/s.

therefore, v_f = 5 m/s

***Illustration* 8**

A block of mass m = 2 kg is attached to a spring whose spring constant is k = 8 N/m. The block slides on an incline for which $\mu_k = \frac{1}{8}$ and □ = 37°. If the block starts at rest with the spring unextended, what is its speed when it has moved a distance d = 0.5m down the incline?



The work done by gravity is positive; the work done by the *spring* and by *friction* are negative.

Solution

This problem can be solved by using Newton's second law. However, the force exerted by the spring varies with position and, therefore, so does the acceleration. We avoid this difficulty by using the work-energy theorem. *(Unless you are solving a problem in static equilibrium, it is a good idea to think of the energy approach rather than dynamics when you see a spring in problem.)*

The work done by the force due to the spring was found in equation.

$x_i = 0$ and $x_f = +d$

The work done by each of the forces on the block are

$W_g = +mgd \sin\theta$

$W_f = \mu_k N d = -\mu_k(mg \cos\theta)d$

$W_s = -\frac{1}{2}kd^2$

Of course, $W_N = 0$. The *work-energy* theorem, with $\Delta K = \frac{1}{2}mv^2 - 0$, tells us

$$mgd \sin\theta - \mu_k(mg \cos\theta)\, d - \frac{1}{2}kd^2 = \frac{1}{2}mv^2$$

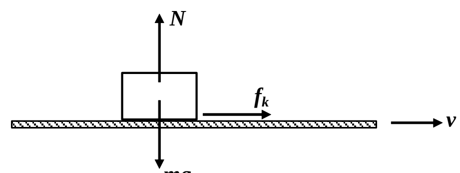
putting $m = 2$ kg; $d = 0.5$ m; $k = 8$ N/m; $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$; $\mu_k = \frac{1}{8}$

we get , $(2)(10)(0.5)(0.6) - \frac{1}{8}(20)(0.8)(0.5) - \frac{1}{2}(8)(0.5)^2 = \frac{1}{2}(2)v^2$

$$v = 2 \text{ m/s}$$

***Illustration* 9**

A box of mass m is gently placed on a conveyor belt that moves at a constant speed v. The coefficient of kinetic friction is μ_k.



A box of mass *m* is gently placed on conveyor belt moving at constant velocity *v*.

(a) What is the work done by friction?

(b) How far does the box move before reaching its final speed?

(c) When the box reaches its final speed, how far has the belt moved?

Solution

(a) When the box is first placed on the belt there will be slipping between the two. But the force of friction on the box and its displacement are in the same direction. Consequently, the work done by kinetic friction is positive. Since the final speed of the box is v,

$$W_f = \Delta K = +\frac{1}{2}mv^2 \qquad \text{(i)}$$

(b) The force of friction is $f = \mu_k N = \mu_k mg$ and $W_f = +fd$.

Thus from equation (i),

$$+\mu_k\, mgd = +\frac{1}{2}mv^2 \qquad \text{(ii)}$$

Thus, $d = \frac{v^2}{2\mu_k g}$

(c) If the box takes a time t to reach speed v, then $v = at$ where a is the acceleration of box.

In this time it will move $d = \frac{1}{2}at^2 = \frac{1}{2}vt$. Since the belt's speed is fixed, in time t it moves a distance $vt = 2d$.

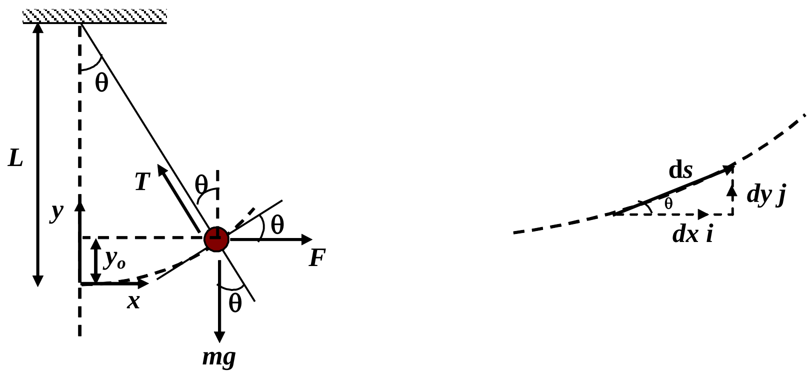
The belt moves *twice* as far as the box while the box is accelerating.

***Illustration* 10**

A horizontal force F very slowly lifts the bob of a simple pendulum from a vertical position to a point at which the string makes an angle θ_o to the vertical. The magnitude of the force is varied so that the bob is essentially in equilibrium all times.

What is the work done by the force on the bob?

Solution



(a) To move the bob at constant speed the force must vary with the angle θ
(b) An infinitesimal displacement has vertical and horizontal components.

Figure is a free body diagram of the system and shows the forces acting on the bob. Since the acceleration is zero, both the vertical and horizontal components of the forces balance:

$$\Sigma F_x = F - T\sin\theta = 0$$

$$\Sigma F_y = T\cos\theta - mg = 0$$

Eliminating T we get

$$F = mg\tan\theta \qquad \text{(i)}$$

This is how the force must vary as a function of angle θ in order for the bob to be in equilibrium.

The work done by $\vec{\mathbf{F}}$ in an infinitesimal displacement $d\vec{\mathbf{s}}$ along the circular arc is

$$dW = \vec{\mathbf{F}}\,.\,d\vec{\mathbf{s}} = F_x\,dx$$

$$= mg\tan\theta\,dx \qquad \text{(ii)}$$

From figure, w see that $\frac{dy}{dx} = \tan\theta$; thus, $dy = \tan\theta\,dx$.

Equation (ii) becomes $dW = mg\,dy$, therefore, the total work done from $y = 0$ to $y = y_o$ is

$$W = \int_0^{y_o} mgdy = mgy_o = mgL(1 - \cos\theta_o)$$

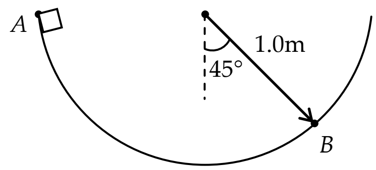
where the vertical displacement is

$y_o = L(1 - \cos\theta_o)$.

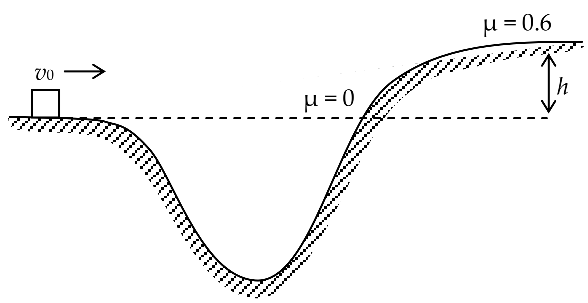
PRACTICE EXERCISE

6. A spring requires 46J of work to extent it 12cm and 270J of work to extend it 27cm. Is the spring force linearly varying with the stretch.

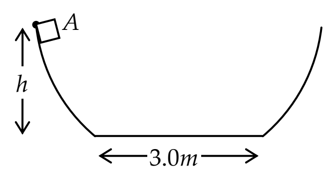
7. A block shown in figure slides on a semicircular frictionless track. If it starts from rest at position *A*, what is its speed at the points marked *B*?



8. In the figure shown, a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless until the block reaches the higher horizontal level. The frictional force in the horizontal track stops the block in a distance *d*. The block's initial speed v_0 is 6 m/s, the height difference *h* is 1.1 m and the coefficient of kinetic friction μ is 0.6 Find *d*.



9. A small particle slides along a track with elevated end and a flat central part, as shown in figure. The flat part has a length 3 m. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic energy is μ = 0.2. The particle is released at point *A*, which is at a height *h* = 1.5m above the flat part of the track. Where does the particle finally come to rest?



Answers

6. No **7.** 3.7 m/s **8.** 1.17m

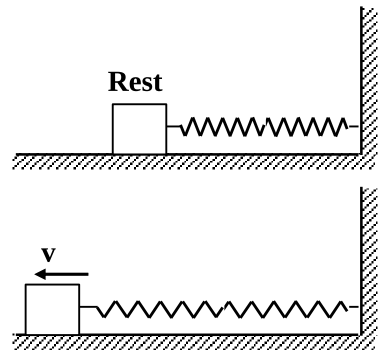
9. mid point of the flat track

5.4 Potential Energy

When we throw a ball upwards with an initial velocity, it rises to a certain height and becomes stationary for a moment. *What happens to the lost kinetic energy?* We know with our experience that the ball returns back in our hands with a speed equal to its initial value. The initial kinetic energy is somehow stored and is later fully recovered in the form of kinetic energy. The ball must have something at the new height that it does not have at the previous level. That something by virtue of its position is *Potential Energy*.

Potential energy is the energy associated with the relative positions of two or more interacting particles.

Potential energy fits well the idea of energy as the capacity to do work. For example, the gravitational potential energy of an object raised off the ground can be used to compress or expand a spring or to lift another weight. As a coil spring unwinds, or a straight spring returns to its natural length, the stored potential energy can be used to do work. For *example*, if a block is attached to a compressed spring, the elastic potential energy can be converted into kinetic energy of the block as shown in figure.



The block gains kinetic energy when the compressed spring is released.

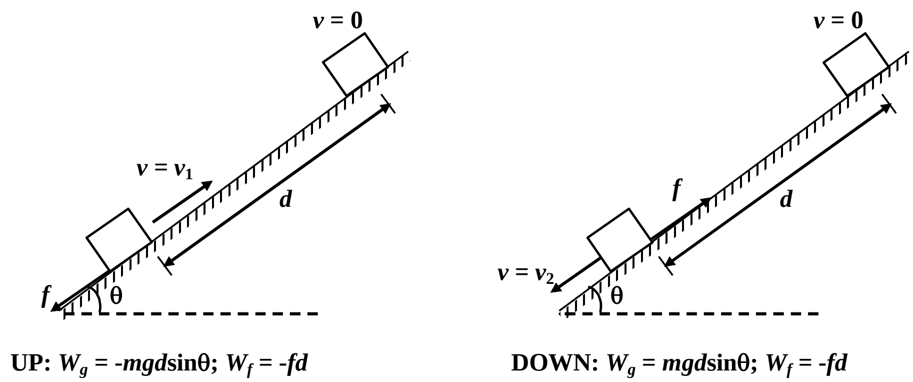
In the above discussion we have seen that in the case of gravity and elastic spring the kinetic energy imparted initially is stored as potential energy for a short time which is regained, later on. But this is *not true* in all cases.

For *example*, consider block placed at rest on a rough horizontal surface. If we impart it some initial kinetic energy, it starts sliding on the surface, the frictional force does negative work on the block, decreasing its kinetic energy to zero. *But it does not come back to our hand no matter how long we wait!* The frictional force has *used up* the kinetic energy in a non - reversible way.

The forces, such as gravity and spring force, which does work in a *reversible* manner is called a *conservative force*. In contrast, the force, such a frictional force, which does work in an irreversible manner is called an *non - conservative force*.

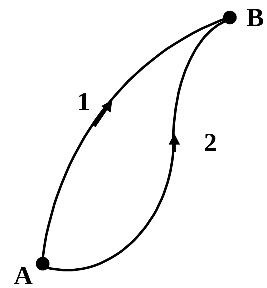
Important

1. The work done by a conservative force is *independent* of path. It depends only on the *initial* and *final* positions. In contrast, the work done by a non-conservative force depends on the path (see figure)



A block slides up and down on a rough inclined plane. In the complete trip, the work done by gravity is zero whereas the work done by friction is negative.

2. The work done by a conservative force around any closed path is zero.



The work done by a conservative force from point A to point B is the same for any two paths such as 1 and 2.

The potential energy is defined only for conservative forces.

The change in potential energy as a particle moves from point A to point B is equal to the negative of the work done by the associated conservative force

$$\Delta U = U_B - U_A = -W_C$$

Using definition of work

$$U_B - U_A = -\int_A^B \mathbf{F}_C . d\mathbf{s} \qquad \ldots (i)$$

From equation, we see that starting with potential energy U_A at point A, we obtain a unique value U_B at point B, because W_C has the same value for all paths. When a block slides along a rough floor, the work done by the force of friction on the block depends on the length of the path taken from point A to point B. There is no unique value for the work done, so one can not assign unique values for potential energy at each point. Hence, non-conservative force can not have potential energy.

When the forces within a system are conservative, external work done on the system is stored as potential energy and is fully recoverable.

Note that the potential energy is *always* defined with respect to a *reference* point.

Gravitational Potential Energy (Near the Earth's Surface)

The work done by gravity on a particle of mass m whose vertical coordinate changes from y_A to y_B is

$$W_g = -mg(y_B - y_A)$$

From equation (i), we have $W_g = -\Delta U_g = -(U_B - U_A)$.

Thus gravitational potential energy at the point B near the surface of the earth is given by

$$U_B = U_A + mgh$$

If we assume potential energy at the point A to be zero, then potential energy at the point B is given by

$$U_B = 0 + mgh = mgh$$

Spring Potential Energy

The work done by the spring force when the displacement of the free end changes from x_i to x_f is given by equation (5.8).

$$W_S = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

By definition $W_S = -\Delta U_S = -(U_f - U_i)$, therefore,

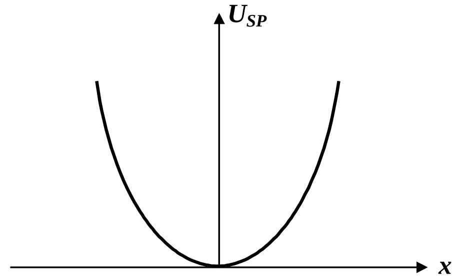
$$U_f = U_i + \frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$$U_S = \frac{1}{2}kx^2$$

If we assume the potential energy stored in the spring at equilibrium is zero and all the displacements are measured from equilibrium, then $x_i = 0$ and $U_i = 0$.

Thus, final energy stored in the spring is

$$U_f = \frac{1}{2}kx^2 \qquad (\because\ x_f = x)$$



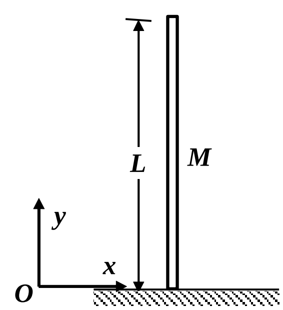
The potential energy of an ideal spring is a parabolic function of the displacement x from equilibrium.

The potential energy function for an ideal spring is a parabolic function as shown in figure.

ILLUSTRATIONS

***Illustration* 11**

A uniform rod of mass M and length L is held vertically upright on a horizontal surface as shown in the figure. Find the potential energy of the rod if the zero potential energy level is assumed at the horizontal surface.



Solution

Since the parts of the rod are at different levels with respect to the horizontal surface, therefore, we have to use the integration to find its potential energy. Consider a small element of length *dy* at a height *y* from the horizontal.

Mass of the element is

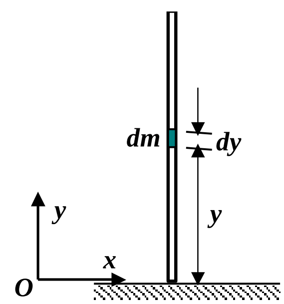
$$dm = \frac{M}{L}dy$$

Its potential energy is given by

$$dU = (dm)gy$$

or $$dU = \frac{M}{L}gydy$$

On integrating, we get



$$U = \frac{Mg}{L}\int_0^L ydy$$

or $$U = \frac{Mg}{L}\left|\frac{y^2}{2}\right|_o^L$$

or $$U = \frac{1}{2}MgL$$

Note that the potential energy of the rod is equal to the product of *Mg* and height of the center of mass $\left(\frac{L}{2}\right)$ from the surface.

5.5 Conservation of Mechanical Energy

From the previous section we know that the work done by a conservative force in terms of the change in potential energy is given by

$$\Delta U = -W_C \qquad \text{(i)}$$

where U is the potential energy and W_C is the work done by a conservative force.

From the *work-energy theorem*, we know that

$$W_{net} = \Delta K$$

Where W_{net} represents the sum of work done by all the forces acting on the mass.

If a particle is subject to only *conservative forces*, then

$$W_C = W_{net} = \Delta K$$

Thus, the equation (i) becomes,

$\Delta U = -\Delta K$

or $\Delta U + \Delta K = 0$ (ii)

The equation (ii) tells us that the *total change in potential energy plus the total change in kinetic energy is zero if only conservative forces are acting on the system*.

That is, there is no change in the sum of K plus U.

□ $\Delta(K + U) = 0$ (iii)

or $\Delta E = 0$ where $E = K + U$

The quantity $E = K + U$ is called the *total mechanical energy*.

According to equation (iii), *when only conservative forces act, the change in total mechanical energy of a system is zero*, in otherwords,

If only conservative forces perform work *on* and *within* a system of masses, *the total mechanical energy of the system is conserved*.

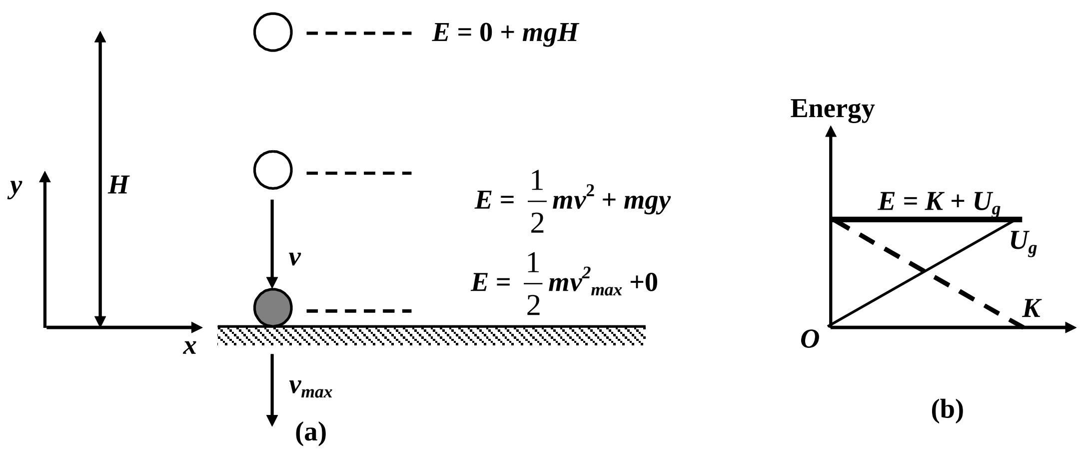
Alternatively, the equation (iii) may be written as

$(K_f + U_f) - (K_i + U_i) = 0$

or $K_f + U_f = K_i + U_i$ (iv)

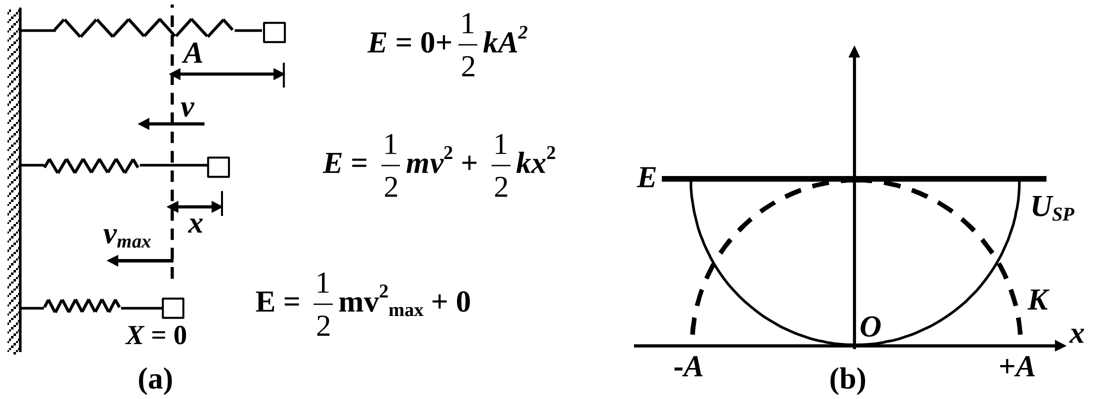
Since $\Delta E = 0$, integrating both sides,

we get E = constant.□



(a) As an object falls from height H, its potential energy is converted to kinetic energy. At height H, the energy is $E = mgH$. Just as it lands, $E = \frac{1}{2}mv_{\max}^2$.

(b) The potential energy and the kinetic energy vary linearly with vertical height y. The mechanical energy $E = K + U = \frac{1}{2}mv^2 + mgy$ stays constant.



(a) A block connected to a spring. At the maximum extension $x = A$, the energy is $E = \frac{1}{2}kA^2$.

When $x = 0$, the energy is $E = \frac{1}{2}mv^2_{max}$

(b) The variation of K and U with x. The energy of the block-spring system is constant.

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 =$ constant.

ILLUSTRATIONS

Illustration 12

A load W is suspended from a self-propelled crane by a cable of length d figure (a). The crane and load are moving at a constant speed v_0. The crane is stopped by a bumper and the load on the cable swings out, as shown in figure (*b*). (a) What is the angle through which the load swings? (b) If the angle is 60° and $d = 5m$, what was the initial speed of the crane?

Solution

(a) The cable does no work on the load, so the load's energy is conserved.

$$K_i + U_i = K_f + U_f \; \frac{1}{2}\frac{W}{g}v_0^2 + 0 = 0 + W(d - d\cos\theta)$$

$$v_0^2 == 2gd(1-\cos\theta) = 4gd\sin^2\frac{\theta}{2} \;\; \theta = 2arc\sin\left(\frac{v_0}{2\sqrt{gd}}\right)$$

(b) $v_0 = 2\sqrt{gd}\sin\frac{\theta}{2} = 2\sqrt{(9.8)(5)}\left(\frac{1}{2}\right) = 7\text{m/s}$

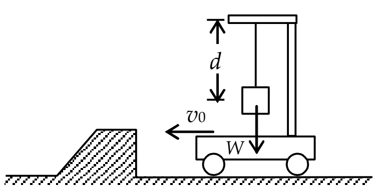


Figure (a)

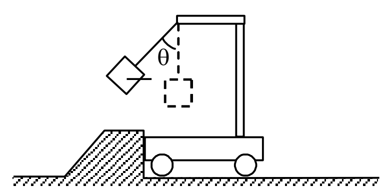
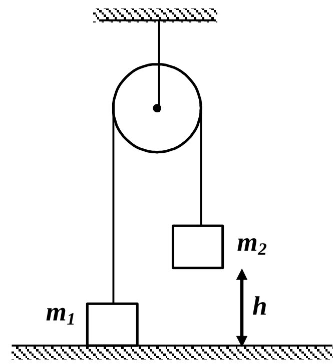


Figure (b)

***Illustration* 13**

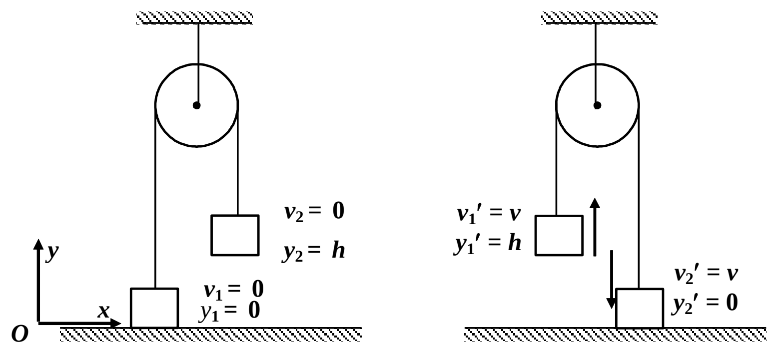
Two blocks with masses m_1 = 3kg and m_2 = 5 kg are connected by a light string that slides over a frictionless pulley as shown in figure. Initially, m_2 is held 5 m off the floor while m_1 is on the floor. The system is then released. At what speed does m_2 hit the floor?



Solution

The initial and final configurations are shown in the figure (b & c).

It is convenient to set $U_g = 0$ at the floor. Initially, only m_2 has potential energy. As it falls, it loses potential energy and gains kinetic energy. At the same time, m_1 gains *potential energy* and *kinetic energy*. Just before m_2 lands, it has only kinetic energy. Let v the final speed of each mass. Then, using the *law of conservation of mechanical energy*.



Initial
(b)

Final
(c)

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}(m_1 + m_2)v^2 + m_1gh = 0 + m_2gh$$

$$v^2 = \frac{2(m_2 - m_1)gh}{m_1 + m_2}$$

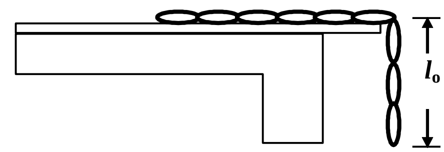
Putting m_1 = 3 kg; m_2 = 5 kg; h = 5 m and g = 10 m/s^2

We get $$v^2 = \frac{2(5-3)(10)(5)}{5+3}$$

or v = 5 m/s.

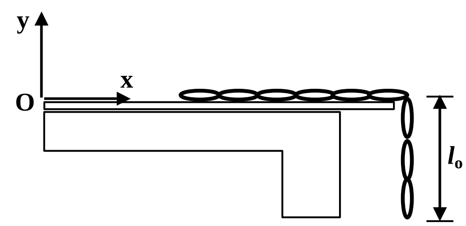
Illustration 14

A chain of length l = 80 cm and mass m = 2 kg is hanging from the end of a plane so that the length l_o of the vertical segment is 50 cm as shown in the figure. The other end of the chain is fixed by a nail. At a certain instant, the nail is pushed out, what is the velocity of the chain at the moment it completely slides off the plane? Neglect the friction.



Solution

We assume the zero potential energy level at the horizontal plane. The *initial* and *final* configuration of the chain are shown in the figure.



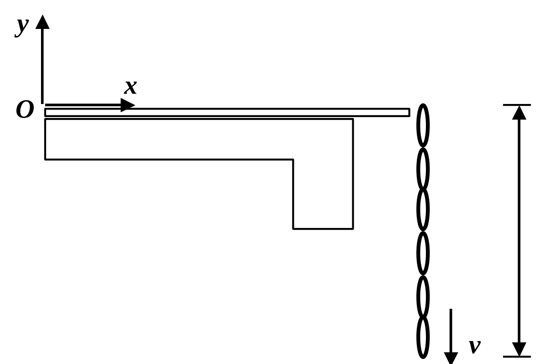
Initial

Initially, $K_i = 0$

$$U_i = 0 + \left(\frac{m}{l} l_o\right) g \left(-\frac{l_o}{2}\right)$$

or $$U_i = -\frac{m l_o^2}{2l} g$$

Note that the part of chain lying over the table has zero potential energy.



Final

Finally, $$K_f = \frac{1}{2} m v^2$$

Where v is the final velocity of the chain.

and $$U_f = -mg\frac{l}{2}$$

Using the *law of energy conservation*

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} m v^2 - mg\frac{l}{2} = 0 - \frac{m l_o^2 g}{2l}$$

or $$v = \sqrt{\frac{g}{l}\left(l^2 - l_o^2\right)}$$

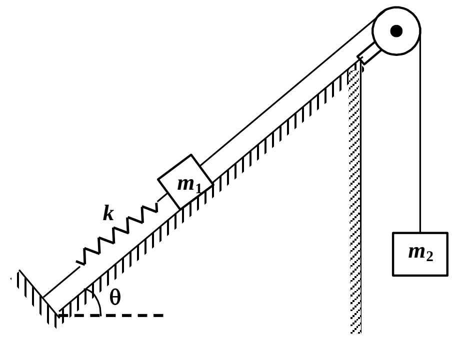
putting l = 0.8 m; l_o = 0.5 m; g = 10 m/s^2, we get

$$v = \sqrt{5} \text{ m/s}$$

or $$v = 2.23 \text{ m/s}$$

Illustration 15

Two blocks with masses m_1 = 2 kg and m_2 = 3 kg hang on either side of a pulley as shown in figure. Block m_1 is on an incline (θ = 30°) and is attached to a spring whose stiffness constant is 40 N/m. The system is released from rest with the spring in its natural length. Find



(a) the maximum extension of the spring

(b) the speed of m_1 when the extension is 0.5 m. Ignore friction and mass of the pulley.

Solution

To use $E_f = E_i$ we would need to assign the initial heights of the blocks arbitrary values h_1 and h_2. The corresponding potential energies, m_1gh_1 and m_2gh_2 would appear in both E_i and E_f and hence would cancel.

We avoid this process by using the form $\Delta K + \Delta U = 0$ instead, since it does not require $U = 0$ reference level.

(a) At the maximum extension x_{max}, the blocks come to rest, and thus $\Delta K = 0$. Next, we must find the changes in U_g and U_S. When m_2 falls by x_{max}, the spring extends by x_{max} and m_1 rises by $x_{max}\sin\theta$. Therefore,

$$\Delta K + \Delta U_g + \Delta U_S = 0$$

$$0 + (-m_2gx_{max} + m_1gx_{max}\sin\theta) + \frac{1}{2}kx_{\max}^2 = 0$$

Thus, $$x_{max} = \frac{2g}{k}\left(m_2 - m_1\sin\theta\right) = 0.98\,\text{m}$$

(b) In this case the change in kinetic energy is $\Delta K = \frac{1}{2}(m_1 + m_2)v^2$.

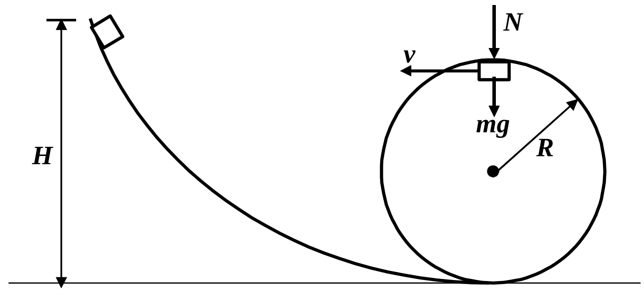
The change in potential energy has the same form as in part (a), but with x_{max} replaced by x = 0.5 m,.

$$\frac{1}{2}\left(m_1 + m_2\right)v^2 + \left(-m_2gx + m_1gx\sin\theta\right) + \frac{1}{2}kx^2 = 0$$

Putting the given values we find v = 1.39 m/s.

***Illustration* 16**

A body slides down an inclined surface which ends into a vertical loop of radius R = 40 cm. What must be the height H of the inclined surface for the body not to fall at the uppermost point of the loop? Assume friction to be absent.



Solution

Let v be the velocity of the particle at the highest point.

According to *Newton's Second Law*, the net force

$F_{net} = N + mg$ provides the centripetal force.

Therefore, $$N + mg = \frac{mv^2}{R}$$

The body is not detached from the loop if $N \geq 0$. In the limiting case, $N = 0$.

That is $$mg = \frac{mv^2}{R} \quad \text{or} \quad v^2 = gR$$

Applying energy conservation at the initial and highest point of the loop, we get

$$mgH = mg(2R) + \frac{1}{2}mv^2$$

Using $v^2 = gR$, we obtain,

$$mgH = mg(2R) + \frac{1}{2}m(gR)$$

or $$H = \frac{5}{2}R = 2.5R$$

Putting R = 40 cm = 0.4 m, we get

H = (2.5) (0.4) = 1m.

5.6 Conservative Forces and Potential Energy Functions

We now consider how we can find a conservative force if we are given the associated potential energy function. According to equation, an infinitesimal change in potential energy dU is related to the work done by the conservative force F_c in an infinitesimal displacement ds as follows

$$dU = -\vec{\mathbf{F}}_{\mathbf{C}} . d\,\vec{\mathbf{s}}$$

In one dimension, the above equation reduces to $dU = -F_x dx$

Thus, $F_x = -\frac{dU}{dx}$

Let us see how equation can be used for the common known cases:

For gravitational potential energy,

$$U_g = mgy \qquad F_y = -\frac{dU}{dy} = -mg$$

For spring potential energy,

$$U_S = \frac{1}{2}kx^2 \qquad F_x = -\frac{dU}{dx} = -kx$$

A conservative force can be derived from a scalar potential energy function

Consider an arbitrary potential energy function $U(r)$ as shown in figure.

The radial component of the associated conservative force is negative of the slope of the potential energy function, that is

$$F_r = -\frac{dU}{dr}$$

The force function may be plotted qualitatively as shown in the figure. If $F_r > 0$, the force is directed toward positive r, which means repulsion, whereas $F_r < 0$ means attraction.

The following important points can be easily noticed by looking at the potential energy and force diagrams.

$(r > r_2)$: $F_r > 0$

The particle is *weakly* repelled.

$(r = r_2)$: $F_r = 0$

At the *maximum* point of the potential energy function, the particle would be in *unstable equilibrium*. If the particle were slightly displaced either to the left or to the right, it would tend to move away from this point.

$(r_o < r < r_2)$: $F_r < 0$

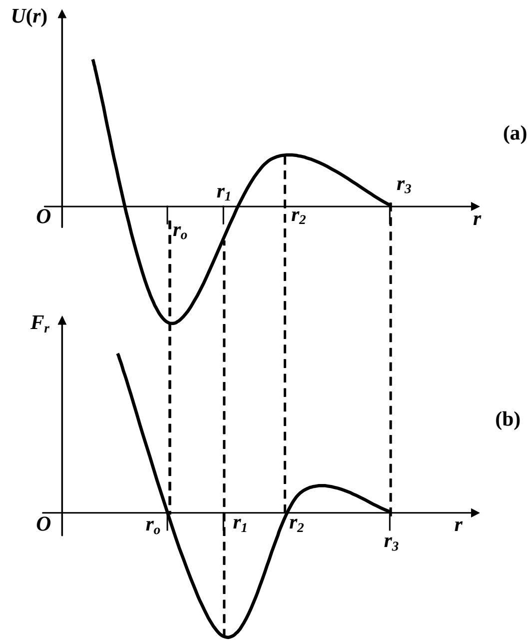
The force is *attractive*, being strongest at r_1 where the slope is greatest.

$(r = r_o)$: $F_r = 0$

At the *minimum* point of the potential energy function, the particle would be in *stable equilibrium*. If slightly displaced in either direction, it would tend to return to this point.

$(r < r_o)$: $F_r > 0$

The particles repel each other. The repulsive force becomes stronger as r is reduced (since the slope of $U(r)$ gets steeper).



From the given potential energy function $U(r)$ one can find the radial component of the force from $F_r = -dU/dr$, which is the negative of the slope of the $U(r)$ curve. A positive force means repulsion, and a negative force means attraction.

ILLUSTRATIONS

***Illustration* 17**

Calculate the forces $F(y)$ associated with the following one-dimensional potential energies:

(a) $U = -$ □y, (b) $U = ay^3 - by^2$, (c) $U = U_0 \sin$ □y

Solution

(a) $F = -\dfrac{dU}{dy} = \omega$

(b) $F = -\dfrac{dU}{dy} = -3ay^2 + 2by$

(c) $F = -\dfrac{dU}{dy} = -\beta U_0 \cos\beta y$

***Illustration* 18**

The potential energy function for the force between two atoms in a diatomic molecule can be expressed approximately as

$$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$$

where a and b are constants and r is the separation between the atoms.

(a) Determine the force function $F(r)$.

(b) Find the value of r for which the molecule will be in the stable equilibrium.

Solution

(a) The force between the two atoms is given by

$$F(r) = -\frac{dU}{dr}$$

or $$F(r) = \frac{12a}{r^{13}} - \frac{6b}{r^7}$$

(b) For stable equilibrium $F(r) = 0$ and $\frac{d^2U}{dr^2} > 0$

Thus, $$+\frac{12a}{r^{13}} - \frac{6b}{r^7} = 0$$

or $$r = \left(\frac{2a}{b}\right)^{1/6}$$

5.7 Power

Power is defined *as the rate at which work is done*. If an amount of work ΔW is done in a time interval Δt, then the average power is defined to be

$$P_{av} = \frac{\Delta W}{\Delta t}$$

The *SI* unit of power is J/s which is given the name watt (W) in the honour of *James Watt*.

Thus, 1 W = 1 J/s.

The *instantaneous* power is the limiting value of P_{av} as $\Delta t \to 0$; that is

$$P = \frac{dW}{dt}$$

The work done by force F on a object that has an infinitesimal displacement $d\vec{\mathbf{s}}$ is dW = $\vec{\mathbf{F}}.d\vec{\mathbf{s}}$ Since the velocity of the object is $\vec{\mathbf{v}} = \frac{d\vec{s}}{dt}$, the instantaneous power may be written as $P = \frac{dW}{dt} = \vec{\mathbf{F}}.\frac{d\vec{s}}{dt}$

or $$P = \vec{\mathbf{F}}.\vec{\mathbf{v}}$$

Since the work and energy are closely related, a more general definition of power is the *rate of energy transfer from one body to another*, or *the rate at which energy is transformed from one form to another*.

$$P = \frac{dE}{dt}$$

ILLUSTRATIONS

***Illustration* 19**

A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_r is varying with time t as $a_r = k^2rt^2$, where k is a constant, What is the power delivered to the particle by the forces acting on it.

Solution

Let v be the instantaneous speed of the particle, then centripetal acceleration is given by

$$a_r = \frac{v^2}{r}$$

Since $a_r = k^2rt^2$ is given, therefore,

$$\frac{v^2}{r} = k^2rt^2$$

or $\quad v = krt$

The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = kr$$

The tangential force is $F_t = ma_t = mkr$

Hence, power delivered is

$$P = F_tv = (mkr)(krt)$$

or $\quad P = mk^2r^2t$

***Illustration* 20**

A small body of mass m is located on a horizontal plane at the point O. The body acquires a horizontal velocity u.

(a) Find the mean power developed by the friction force during the whole time of motion.

(b) Find the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $k = \alpha x$, where α is a constant and x is the distance from the point O.

Solution

(a) The frictional force acting on the body $\quad f_r = \mu mg$

Retardation provided by this force is $\quad a = -\mu g$

Total time taken by the body to come to rest is

$$t = \frac{u}{a}$$

$$= \frac{u}{\mu g}$$

Total change in kinetic energy due to friction is

$$\Delta E = \frac{1}{2}\text{mu}^2 - 0$$

Mean power can be given as

$$= \frac{\text{Net gain in kinetic energy}}{\text{Total time of motion}} = \frac{\Delta E}{t}$$

or $$\frac{\frac{1}{2}mu^2}{u/\mu g} = \frac{1}{2}mu\mu g$$

(b) When friction coefficient is k = αx, the friction force on the body when it is at a distance x from the point O is

$$f_r = \alpha xmg$$

Retardation due to this force is α = – αgx

or $$v\frac{dv}{dx} = -\alpha gx$$

or $$vdv = -\alpha gx$$

Integrating the a above expression for velocity at a distance x from point), gives

$$\int_u^v v\,dv = -\int_{uv} \alpha gx\,dx$$

$$v^2 = u^2 - \alpha gx^2$$

Instantaneous power due to friction force at a distance x from point O is

$$P = F.v$$

or $$= -\alpha mgx\sqrt{(u^2 - \alpha gx^2)}$$

This power is maximum when $\frac{dp}{dx} = \frac{\alpha mgx}{\left[u^2 - \alpha gx^2\right]^{1/2}} \times \alpha gx - \alpha mg[u^2 - \alpha gx^2]^{1/2} = 0$

or $$x = \frac{u}{\sqrt{2\alpha g}}$$

PRACTICE EXERCISE

10. A body of mass m is thrown at an angle θ to the horizontal with the initial velocity *u*. Find the mean power developed by gravity over the whole time of motion of the body, and the instantaneous power of gravity as a function of time.

11. A pump is required to lift 1000 kg of water power minute from a well 12 m deep and eject it with a speed of 20 m/s. How much work is done per second in lifting the water and what must be the power output of the pump?

Answers

10. Zero, P = mg (gt – usin□) **11.** 5333.33 J/s

*****

MISCELLANEOUS PROBLEMS

OBJECTIVE TYPE

Example 1

When a body is moving in a horizontal circular path with a fixed speed, which one of the following physical quantities remains constant?

(a) Momentum of the body

(b) Centripetal acceleration

(c) Kinetic energy of the body

(d) Displacement of the body from the centre of the circular path

Solution

Kinetic energy remains constant as mass and magnitude of velocity remain constant. **Ans. (c)**

Example 2

A body of mass m is moving in a circle of radius r with a constant speed v. The net force on the body is mv^2/r and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle?

(a) $\frac{mv^2}{r} \times \pi r$ (b) Zero (c) $\frac{mv^2}{r^2}$ (d) $\frac{\pi r^2}{mv^2}$

Solution

Since the net force acting on the body is always perpendicular to the displacement, therefore, the net work done on the particle is zero. **Ans. (b)**

Example 3

A chain of mass m is placed on a smooth table with $1/7^{th}$ of its length l hanging over the edge. The work done in pulling the chain back to the surface of table is

(a) $mg\frac{l}{7}$ (b) $mg\frac{l}{14}$ (c) $mg\frac{l}{49}$ (d) $mg\frac{l}{98}$

Solution

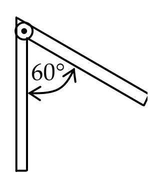
The work done is equal to the increase in potential energy of the chain.

The initial potential energy the chain is

$$U_i = -\frac{m}{7} g \frac{l}{14} = -\frac{mgl}{98}$$ **Ans. (d)**

Example 4

A metre stick, of mass 600 gram, is pivoted at one end and displaced through an angle of 60°. The increase in its potential energy is ($g = 10\ ms^{-2}$)



(a) 1.5 J (b)

(c) 150 J (d)

Solution

Increase in potential energy = mg (height through which C.G. of stick rises)

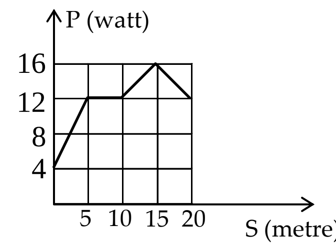
$= mg\left(\frac{1}{2}\right)(1 - \cos\theta)$

$= \frac{600}{1000}(10)\left(\frac{1}{2}\right)(1-\cos\theta)$

$= 0.6\,(10)\,\frac{1}{4} = 1.5$ J **Ans. (a)**

Example 5

The figure here shows the frictional force versus displacement for a particle in motion. The loss of kinetic energy (work done against friction) in traveling over S = 0 to 20 m will be



(a) 80 J (b) 160 J

(c) 240 J (d) 24 J

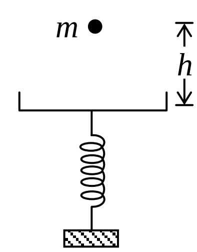
Solution

Area of force vs displacement graph with s-axis gives work done

Area = 240; W = 240 J **Ans. (c)**

Example 6

A ball of mass m is dropped from a height h on a platform fixed at the top of a vertical spring. He platform is displaced by a distance x. The spring constant is



(a) $\frac{2mg}{x}$ (b) $\frac{2mgh}{x^2}$

(c) $\frac{2mg(h+x)}{x^2}$ (d) $\frac{2mg(h+x)}{h^2}$

Solution

KE of moving mass is converted into elastic potential energy of spring

$\therefore \quad \frac{1}{2}\times mv^2 = \frac{1}{2}kx^2$

$\frac{1}{2}\times 0.5\times(1.5)^2 = \frac{1}{2}\times 50x^2$

or $\quad x^2 = \frac{1}{2}\times\frac{(1.5)^2}{50}$, $\quad x = \frac{1.5}{10} = 0.15m$ **Ans. (c)**

SUBJECTIVE TYPE

Example 1

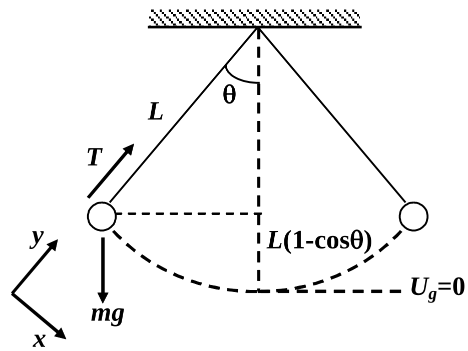
The bob of a simple pendulum of length L = 2 m has a mass m = 2 kg and a speed v = 1 m/s when the string is at 35° to the vertical. Find the tension in the string at

(a) the lowest point in its swing

(b) the highest point

Solution

The problem requires to use dynamics and the conservation of mechanical energy. The forces on the bob are shown in the figure.



The acceleration has both *radial* and *tangential* components.

The equation for tangential component,

$$\Sigma F_x = mg \sin\theta = ma$$

Since the bob is moving in a circular path of radius L, the equation for the radial component is

$$\Sigma F_y = T - mg\cos\theta = \frac{mv^2}{L} \qquad \text{(i)}$$

To find the tension we need the speed, which can be found from the conservation law. We set U_g = 0 at the lowest point.

Note that the height is $y = L - L\cos\theta$. The mechanical energy is

$$E = \frac{1}{2}mv^2 + mgL(1-\cos\theta) \qquad \text{(ii)}$$

$$= \frac{1}{2}(2\text{kg})(1\text{ m/s})^2 + (2\text{kg})(9.8\text{ N/kg})(2\text{m})(1-0.8) = 9\text{ J}$$

(a) At the lowest point $\theta = 0$; hence (ii) becomes

$$E = \frac{1}{2}mv_{\max}^2 + 0$$

Since E = 9 J we find v_{max} = 3 m/s.

Now we have the speed we can find the tension at $\theta = 0$. From (i), $T - mg = \frac{mv_{\max}^2}{L}$, from which we find

$$T = 20 + 9 = 29\text{ N}$$

(b) At the highest point v = 0; hence, (ii) becomes

$$E = 0 + mgL(1 - \cos\theta_{max})$$

Using $E = 9$ J lead to $\cos\theta_{max} = \frac{31}{40}$

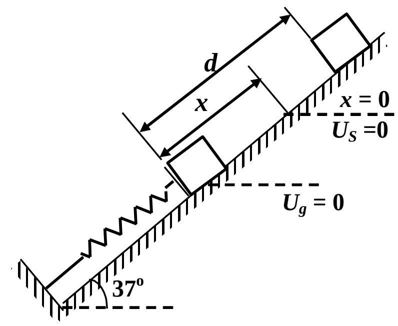
Since $v = 0$, therefore, $T = mg\cos\theta_{max} = 15.5$ N

Example 2

A block of mass $m = 0.2$ kg is held against, but not attached to a spring ($k = 50$ N/m) which is compressed by 20 cm, as shown in figure, when the block is released, the block slides 50 cm up the rough incline before coming to rest. Find

(a) the force of friction

(b) the speed of the block as it leaves the spring.



Solution

We could set $U_g = 0$ at $x = 0$, but if we use the lowest point instead, all subsequent values are positive. Again the total energy has three terms $E = K + U_g + U_S$.

(a) We set $x_o = 0.2$ m and $d = 0.5$ m. Both K_i and K_f are zero, so $E_i = \frac{1}{2}kx_o^2$ and $E_f = mgd\sin\theta$. From work-energy theorem,

$$E_f - E_i = W_{nc}$$

$$mgd\sin\theta - \frac{1}{2}kx_o^2 = -fd$$

Thus $f = 0.82$ N

(b) The initial energy E_i is the same as above, but the final value at $x = 0$ is

$$E_f = \frac{1}{2}mv^2 + mgx_o\sin\theta$$

From work-energy theorem,

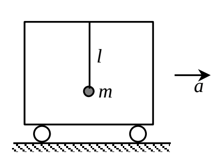
$$E_f - E_i = W_{nc}$$

$$\frac{1}{2}mv^2 + mgx_o\sin\theta - \frac{1}{2}kx_o^2 = -fx_o$$

After solving we get, $v = 2.45$ m/s.

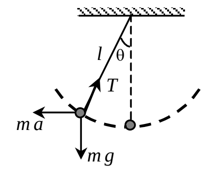
Example 3

A pendulum of mass m and length l is suspended from the ceiling of a trolley which has a constant acceleration a in the horizontal direction as shown in the figure. Find the maximum deflection θ of the pendulum from the vertical.



Solution

The free body diagram of the pendulum with respect to trolley is shown in the figure. The forces acting on the bob are:



the gravity, mg, the pseudo force, ma, the tension, T

The work done by gravity is $W_g = -mgl(1-\cos\theta)$

The work done by pseudo force is $F_{PS} = mal\sin\theta$

The work done by tension is $W_T = 0 \quad (\because T \perp ds)$

At the position of maximum deflection the velocity of the bob is zero, therefore, $\Delta K = 0$.

Applying *Work-Energy Theorem*, we get

$$W_g + W_{PS} + W_T = \Delta K$$

$$-mgl(1-\cos\theta) + mal\sin\theta + 0 = 0$$

or $$g\left[2\sin^2\frac{\theta}{2}\right] = a\left[2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

or $$\tan\frac{\theta}{2} = \frac{a}{g}$$ or $$\square = 2\tan^{-1}\left(\frac{a}{g}\right)$$

Note that this angle is *double* to that at the equilibrium which is $\theta_o = \tan^{-1}\left(\frac{a}{g}\right)$.

Example 4

A small body of mass m is located on a horizontal plane at the point O. The body acquires a horizontal velocity u. Find,

(a) Find the mean power developed by the friction force during the whole time of motion, if the friction coefficient is μ.

(b) Find the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $\mu = \alpha x$, where α is a constant and x is the distance from the point O.

Solution

(a) The frictional force acting on the body

$$f_r = \mu mg$$

Retardation provided by this force is

$$a = -\mu g$$

Total time taken by the body to come to rest is

$$t = \frac{u}{a} = \frac{u}{\mu g}$$

Total change in kinetic energy due to friction is

$$\Delta E = \frac{1}{2}mu^2 - 0$$

Mean power can be given as

$$\text{Mean power} = \frac{\text{Net gain in kinetic energy}}{\text{Total time of motion}} = \frac{\Delta E}{t}$$

$$\Rightarrow \quad = \frac{\frac{1}{2}mu^2}{u/\mu g} = \frac{1}{2}mu\mu g$$

(b) When friction coefficient is $\mu = \alpha x$, the friction force on the body when it is at a distance x from the point O is

$$f_r = \alpha x m g$$

Retardation due to this force is

$$a = -\alpha g x$$

or $$v\frac{dv}{dx} = -\alpha g x$$

or $$vdv = -\alpha g x \, dx$$

Integrating the above expression for velocity at a distance x from point O, gives

$$\int_u^v v \, dv = \int_0^x \alpha g x \, dx$$

$$v^2 = u^2 - \alpha g x^2$$

Instantaneous power due to friction force at a distance x from point O is

$$P = F.v$$

or $$P = -\alpha m g x\sqrt{\left(u^2 - \alpha g x^2\right)} \qquad \ldots (1)$$

This power is maximum when $\frac{dP}{dx} = 0$, thus

$$\frac{dP}{dx} = \frac{\alpha m g x}{\left[u^2 - \alpha g x^2\right]^{1/2}} \times \alpha g x - \alpha m g\left[u^2 - \alpha g x^2\right]^{1/2} = 0$$

or $$x = \frac{u}{\sqrt{2\alpha g}} \qquad \ldots (2)$$

Equation (2) gives the value of x at which instantaneous power is maximum. Using above value of x in equation (1) gives the maximum instantaneous power as

$$P_{\max} = -\alpha m g \frac{u}{\sqrt{2\alpha g}}\sqrt{u^2 - u^2/2}$$

$$P_{\max} = -\frac{1}{2}mu^2\sqrt{\alpha g}$$

Example 5

A small body of mass m is located on a horizontal plane at the point O. The body acquires a horizontal velocity u. Find,

(a) Find the mean power developed by the friction force during the whole time of motion, if the friction coefficient is μ.

(b) Find the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $\mu = \alpha x$, where α is a constant and x is the distance from the point O.

Solution

(a) The frictional force acting on the body

$$f_r = \mu mg$$

Retardation provided by this force is

$$a = -\mu g$$

Total time taken by the body to come to rest is

$$t = \frac{u}{a} = \frac{u}{\mu g}$$

Total change in kinetic energy due to friction is

$$\Delta E = \frac{1}{2}mu^2 - 0$$

Mean power can be given as

$$\text{Mean power} = \frac{\text{Net gain in kinetic energy}}{\text{Total time of motion}} = \frac{\Delta E}{t}$$

$$\Rightarrow \quad = \frac{\frac{1}{2}mu^2}{u/\mu g} = \frac{1}{2}mu\mu g$$

(b) When friction coefficient is $\mu = \alpha x$, the friction force on the body when it is at a distance x from the point O is

$$f_r = \alpha xmg$$

Retardation due to this force is

$$a = -\alpha gx$$

or $$v\frac{dv}{dx} = -\alpha gx$$

or $$vdv = -\alpha gx\ dx$$

Integrating the above expression for velocity at a distance x from point O, gives

$$\int_u^v v\ dv = \int_0^x \alpha gx\ dx$$

$$v^2 = u^2 - \alpha gx^2$$

Instantaneous power due to friction force at a distance x from point O is

$$P = F.v$$

or $$P = -\alpha mgx\sqrt{\left(u^2 - \alpha gx^2\right)} \qquad \ldots (1)$$

This power is maximum when $\frac{dP}{dx} = 0$, thus

$$\frac{dP}{dx} = \frac{\alpha mgx}{\left[u^2 - \alpha gx^2\right]^{1/2}} \times \alpha gx - \alpha mg\left[u^2 - \alpha gx^2\right]^{1/2} = 0$$

or $$x = \frac{u}{\sqrt{2\alpha g}} \qquad \ldots (2)$$

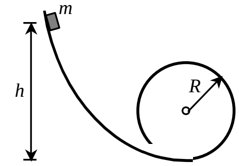
Equation (2) gives the value of x at which instantaneous power is maximum. Using above value of x in equation (1) gives the maximum instantaneous power as

$$P_{\max} = -\alpha mg \frac{u}{\sqrt{2\alpha g}} \sqrt{u^2 - u^2/2}$$

$$P_{\max} = -\frac{1}{2} mu^2 \sqrt{\alpha g}$$

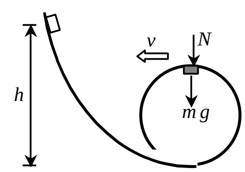
Example 6

A particle of mass m slides down a smooth inclined surface, which ends into a vertical loop of radius R. From what minimum height should the particle be released so that it does not fall at the uppermost point of the loop?



Solution

Let *v* be the velocity of the particle at the highest position. The reference level for potential energy is assumed at the lowest level of the track. Applying energy conservation, we get



$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \frac{1}{2} mv^2 + mg(2R) \qquad (1)$$

Two forces mg and N acts on the block in vertically downward direction.

Applying Newton's second law, $mg + N = \frac{mv^2}{R}$

The particle will reach the highest position if $N \geq 0$. In the liming case $N = 0$. Thus,

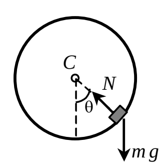
$$mg = \frac{mv^2}{R} \quad \text{or} \quad v^2 = gR$$

Substituting the value of v^2 in the equation (1)

$$mgh = \frac{1}{2} m(ghR) + mg(2R) \qquad \text{or} \qquad h = \frac{5}{2} R$$

Let us verify that the *minimum* value of N occurs at the highest position.

The figure shows the instantaneous position of the particle at which it makes an angle θ with the vertical.



If u be the speed of the particle at this instant, then

$$N - mg\cos\theta = \frac{mu^2}{R} \quad (2)$$

Applying energy conservation, we get

$$mgh = mgR(1 - \cos\theta) + \frac{1}{2} mu^2$$

or $$\frac{mu^2}{R} = 2mg\frac{h}{R} - 2mg(1 - \cos\theta)$$

Substituting this in equation (2), we get

$$N = 2mg\left(\frac{h}{R} - 1\right) + 3mg\cos\theta$$

The following points can be easily noted from the above equation.

- The *minimum* value of N occurs at the highest position i.e. when $\theta = \pi$.
- The particle has the possibility to fall off the track only when $\theta > \pi/2$, and the particle falls

off the track if the height of release h is given by $R < h < \frac{5}{2}R$

Example 7

A particle of mass m moves along a circle of radius R with a normal acceleration varying with time as $a_n = kt^2$, where k is a constant.

(a) Find the tangential force acting on the particle.

(b) Find the net force acting on the particle.

(c) Find time dependence of power developed by all the forces acting on the particle and the mean value of this power averaged over the first t second after the beginning of the motion.

Solution

Normal acceleration $a_n = kt^2$

$$\Rightarrow \qquad \frac{v^2}{R} = kt^2$$

$$\Rightarrow \qquad v = t\sqrt{kR}$$

Now tangential acceleration $a_t = \frac{dv}{dt} = \sqrt{kR}$.

(a) Tangential force acting on the particle,

$$F_t = ma_t = m\sqrt{kR} .$$

(b) Net force, $F = \sqrt{F_t^2 + F_N^2}$

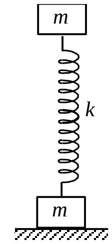
$$= m\sqrt{a_t^2 + a_N^2} = m\sqrt{kR + k^2t^4} .$$

(c) Power developed by normal force is zero where power developed by tangential force is given by

$$P = F_t v = m\sqrt{kR}\ \sqrt{kR}t = mkRt$$

Average power, $< P >= \frac{\int_0^t P dt}{\int_0^t dt} = \frac{\int_0^t mkRt\, dt}{\int_0^t dt} = \frac{mkR\frac{t^2}{2}}{t} = \frac{1}{2}mkRt$

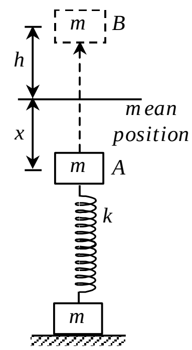
Example 8



A system consists of two identical blocks, each of mass m, linked together by the compressed weightless spring of stiffness k as shown in figure. The blocks are connected by a thread which is burned through at a certain moment. Find at which values of initial compression, the lower block will bounce up after the thread has been burned through.

Solution

Let the initial compression be x in the spring from its natural length. When the thread is burned, the spring shoots towards its natural length and moves up further to a distance h, as shown in figure. This h should be at least equal to that extension in the spring, which is just sufficient to break off the lower mass from ground. When the upper block reaches the point B, the restoring force on lower block will balance its weight. Thus,



$$kh = mg$$

Applying work–energy theorem between points A and B, we get

$$0 + \frac{1}{2}kx^2 - mg(x+h) - \frac{1}{2}kh^2 = 0$$

As upper block comes to rest at B, we take $K.E.$ of block zero at B.

or $\quad x^2 - \frac{2mg}{k}x - \frac{3m^2g^2}{k} = 0 \qquad$ [substituting $h = \frac{mg}{k}$]

Solving, we get

$$x = \frac{3mg}{k} \quad \text{or} \quad -\frac{mg}{k}$$

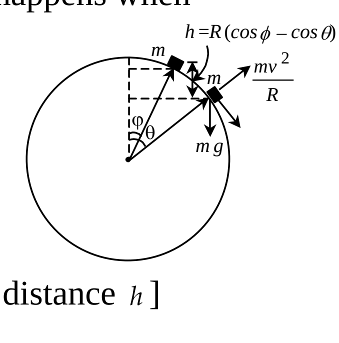
Since x is positive, the minimum initial compression required is $= \frac{3mg}{k}$

Example 9

A small box of mass m is placed on the outer surface of a smooth fixed sphere of radius R at a point where the radius makes an angle ϕ with the vertical. If the box is released from this position, find the distance traveled by the box before it leaves contact with the sphere.

Solution

The situation is shown in figure. It starts falling along the circular path outside the sphere and breaks off from the surface when its contact reaction becomes zero. It happens when



$$\frac{mv^2}{R} = mg\cos\theta$$

$$v = \sqrt{Rg\cos\theta} \qquad \text{… (i)}$$

where v is the instantaneous velocity of the box at an angle θ from the vertical. It can also be obtained by

$$v = \sqrt{2gh} \qquad \text{[as from rest in falls a distance } h\text{]}$$

or $$= \sqrt{2gR(\cos\phi - \cos\theta)} \qquad \text{… (ii)}$$

Using (i) and (ii) equation, we get

$$\sqrt{Rg\cos\theta} = \sqrt{2gR(\cos\phi - \cos\theta)}$$

or $$3\cos\theta = 2\cos\phi$$

or $$\theta = \cos^{-1}\left(\frac{2}{3}\cos\phi\right)$$

Thus distance traveled by the box before leaving the contact with the sphere is

$$s = R(\theta - \phi)$$

or $$= R\left[\cos^{-1}\left(\frac{2}{3}\cos\phi\right) - \phi\right]$$

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Exercise - I

OBJECTIVE TYPE QUESTIONS

Multiple choice questions with ONE option correct

1. The total work done on a particle is equal to the change in its kinetic energy
   (a) always
   (b) only if the forces acting on the body are conservative
   (c) only if the forces acting on the body are gravitational
   (d) only if the forces acting on the body are elastic

2. When the force retards the motion of a body, the work done by the force is
   (a) zero
   (b) negative
   (c) positive
   (d) positive or negative depending upon the magnitude of force and displacement.

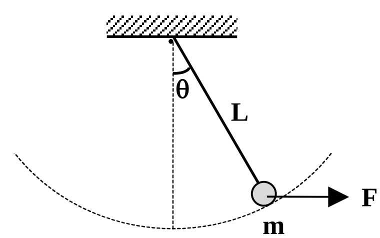
3. A body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done is 25 J, the angle which the force makes with the direction of motion of the body is
   (a) 0° (b) 30° (c) 60° (d) 90°

4. A particle moves along the $x-$axis from $x=0$ to $x=5m$ under the influence of a force given by $F=7-2x+3x^2$. The work done in the process is
   (a) 107 (b) 270 (c) 100 (d) 135

5. When work done by force of gravity is negative
   (a) Potential energy increases (b) Kinetic energy decreases
   (c) Potential energy remains constant (d) Potential energy decreases

6. When work done by a force on a particle is positive
   (a) Kinetic energy may increase (b) Kinetic energy may decrease
   (c) Kinetic energy may be constant (d) All of these

7. An object of mass *m* is tied to a string of length *L* and a variable horizontal force is applied on it which starts at zero and gradually increases until the string makes an angle θ with the vertical . Work done by the force *F* is



   (a) *mgL*(1 - sinθ) (b) *mgL*
   (c) *mgL*(1 - cosθ) (d) *mgL*(1 + cosθ)

8. A chain is held on a frictionless table with one third of its length hanging over the edge. If the chain has a length L and mass M, how much work is required to pull the hanging part back on the table

(a) mgL (b) $\frac{MgL}{3}$ (c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

9. A person pulls a bucket of water from a well of depth h. If the mass of uniform rope is m and that of the bucket full of water M, the work done by the person is

(a) $\left(M+\frac{m}{2}\right)gh$ (b) $\frac{1}{2}(M+m)gh$ (c) $\left(M+\frac{m}{2}\right)gh$ (d) $\left(\frac{M}{2}+m\right)gh$

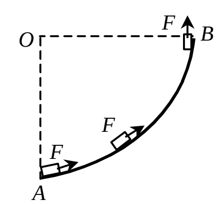
10. A cord is used to lower vertically a block of mass M a distance d at a constant downward acceleration of $g/4$. Then the work done by the cord on the block is

(a) $\frac{Mgd}{4}$ (b) $-\frac{Mgd}{4}$ (c) $\frac{3Mgd}{4}$ (d) $\frac{-3Mgd}{4}$

11. If a simple pendulum of length l has maximum angular displacement θ then the maximum kinetic energy of its bob of mass m is

(a) $\frac{1}{2}m\left(\frac{l}{g}\right)$ (b) $\left(\frac{1}{2}\right)m\left(\frac{g}{l}\right)$ (c) $mgl(1-\cos\theta)$ (d) $\left(\frac{1}{2}\right)m\left(\frac{g}{l}\right)$

12. If the block is pulled by a force F which is always tangential to the surface then, the work done by this force between A and B is



(a) $F\pi R$ (b) $\sqrt{2}FR$

(c) $\frac{\pi}{2}FR$ (d) FR

13. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the longer piece will have a force-constant of

(a) $(2/3)k$ (b) $(3/2)k$ (c) $3k$ (d) $6k$

14. A long spring is stretched by x its PE is U. If the spring is stretched by Nx the PE stored in it will be

(a) $\frac{U}{N}$ (b) NU (c) N^2U (d) $\frac{U}{N^3}$

15. A particle moves with a velocity $\vec{v} = 5\hat{i}-3\hat{j}+6\hat{k}$ m/s under the influence of a constant force $\vec{F}=10\hat{i}+10\hat{j}+20\hat{k}$ N. The instantaneous power applied to the particle is

(a) 200 J/s (b) 140 J/s (c) 40 J/s (d) 170 J/s

16. A body of mass m is projected at an angle θ with the horizontal with an initial velocity u. The mean power of gravity over the whole time of journey is

(a) $mg\cos\theta$ (b) $\frac{1}{2}mg\sqrt{u\cos\theta}$ (c) $\frac{1}{2}mgu\sin\theta$ (d) zero

17. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m (g = 10m/s^2)

(a) 4 s (b) 5 s (c) 8s (d) 10 s

18. A block of mass m is pulled by a constant power P placed on a rough horizontal plane. The friction coefficient between the block and the surface is μ. Maximum velocity of the block will be

(a) $\mu P / mg$ (b) $\mu mg / P$ (c) μmgP (d) $P / \mu mg$

19. A ball is released from the top of a tower. the ratio of work done by force of gravity in first, second and third second of the motion of ball is

(a) $1:2:3$ (b) $1:4:16$ (c) $1:3:5$ (d) $1:9:25$

20. The potential energy function for the force between two atoms of a diatomic molecule is given by $U(x) = a/x^{12} - b/x^6$, where a and b are positive constant and x is the distance between the atoms. The minimum energy needed to break up the molecule is

(a) $2a^2/b$ (b) $b/2a^2$ (c) $b^2/4a$ (d) $-b^2/4a$

21. A particle of mass m is fixed to one end of a light rigid rod of length l and rotated in a vertical circular path about its other end. The minimum speed of the particle at its highest point must be

(a) zero (b) $\sqrt{gl}$ (c) $\sqrt{1.5gl}$ (d) $\sqrt{2gl}$

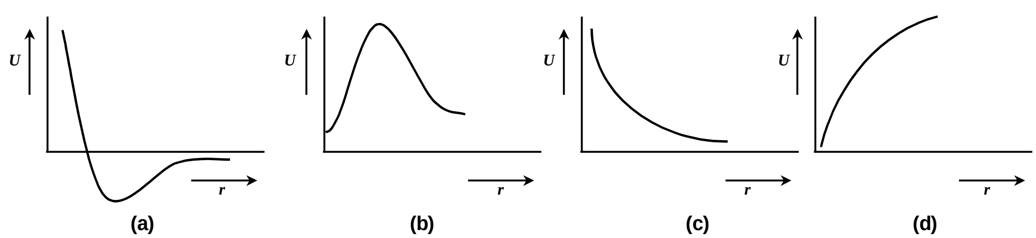
22. If the kinetic energy of a body is increased by 300%, its momentum will increase by

(a) 100% (b) 150% (c) 200% (d) 400%

23. Under the action of a force, a 2 kg body moves such that its position x as a function of time is given by $x = \frac{t^3}{3}$; where x is in meter and t is in second, the work done by the force in the first two second is

(a) 1600 J (b) 160 J (c) 16 J (d) 1.6 J

24. The diagrams in the following figures represents the potential energy U of a diatomic molecule as a function of the inter-atomic distance r. Which corresponds to stable molecules found in nature:



25. A force $\vec{\mathbf{F}} = -k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where k is a positive constant, acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive $x-$axis to the point $(a, 0)$, and then parallel to the $y-$axis to the point (a, a).

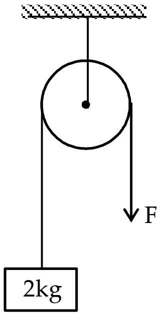
The total work done by the force on the particle is

(a) $-2ka^2$ (b) $2ka^2$ (c) $-ka^2$ (d) ka^2

Multiple choice questions with MORE THAN ONE option correct

1. Work done by a force on an object is zero, if:
   (a) the force is always perpendicular to its acceleration
   (b) the object is stationary but the point of application of the force moves on the object
   (c) the force is always perpendicular to its velocity
   (d) the object moves in such a way that the point of application of the force remains fixed

2. A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F = 40 N. The kinetic energy of the particle increases 40 J in a given interval of time. Then (g = 10 m/s^2)

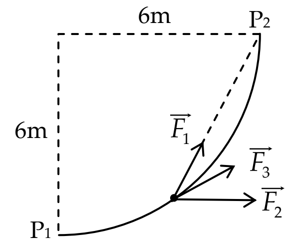


   (a) tension in the string is 40 N
   (b) displacement of the block in the given interval of time is 2m
   (c) work done by gravity is – 20 J
   (d) work done by tension is 80 J

3. A particle moves in a straight line with constant acceleration under a constant force F. Select the correct alternative (s)
   (a) Power developed by this force varies linearly with time
   (b) Power developed by this force varies parabolically with time
   (c) Power developed by this force varies linearly with displacement
   (d) Power developed by this force varies parabolically with displacement

4. One end of a light spring of force constant *k* is fixed to a wall and other end is tied to a block placed on a smooth horizontal surface. For displacement *x*, the work done by the spring is $\frac{1}{2}kx^2$. The possible case (s) may be
   (a) the spring was initially stretched by a distance *x* and finally was in its natural length
   (b) the spring was initially in its natural length and finally it was compressed by a distance *x*
   (c) the spring was initially compressed by a distance *x* and finally was in its natural length
   (d) the spring was initially in its natural length and finally stretched by a distance *x*

5. A smooth track in the form of a quarter circle of radius 6 m lies in a vertical plane. A particle moves from P_1 to P_2 under the action of force $\vec{F_1}, \vec{F_2}$ and $\vec{F_3}$. Force $\vec{F_1}$ is always towards P_2 and is always 20N in magnitude force $\vec{F_2}$ always acts horizontally and is always 30N in magnitude. Force $\vec{F_3}$ always acts tangentially to the track and is of magnitude 15N. Select the correct alternative (s)



(a) Work done by $\vec{F_1}$ is 120 J

(b) Work done by $\vec{F_2}$ is 180 J

(c) Work done by $\vec{F_3}$ is 45 □

(d) $\vec{F_1}$ is conservative in nature

6. A particle is acted upon by a conservative force $\vec{F} = (7\hat{i} - 6\hat{j})N$ (no other force is acting on the particle). Under the influence of this force particle moves from (0, 0) to (– 3m, 4m) then:

(a) work done by the force is 3 J

(b) work done by the force is – 45 J

(c) at (0, 0) speed of the particle must be zero

(d) at (0, 0) speed of the particle may or may not be zero

7. In the pulley-block system shown in figure strings are light. Pulleys are massless and smooth. System is released from rest. In 0.3 second

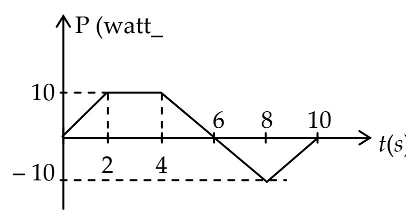
(a) work done on 2 kg block by gravity is 6 J

(b) work done on 2 kg block by string is – 2 J

(c) work don on 1 kg block by gravity is – 1.5 J

(d) work done on 1 kg block by string is 2 J

8. Power of a force acting on block varies with time t as shown in figure. Then angle between force acting on the block and its velocity is:



(a) acute at t = 1s

(b) 90° at t = 3s

(c) obtuse at t = 7s

(d) change in kinetic energy from t = 0, to t =10s is 20J

9. The potential energy U of a particle of mass 1kg moving in *x*-*y* plane obeys the law $U = 3x + 4y$, where (x, y) are the co-ordinates of the particle in metre. If the particle is at rest at (6, 4) at time $t = 0$ then

(a) the particle has constant acceleration

(b) the particle has zero acceleration

(c) the speed of particle when it crosses the *y*-axis 10 m/s

(d) co-ordinate of particle at $t = 1$s are (4.5, 2)

10. A small spherical ball is suspended through a string of length *l*. The whole arrangement is placed in a vehicle which is moving with velocity *v*. Now suddenly the vehicle stops and balls starts moving along a circular path. If tension in the string at the highest point is twice the weight of the ball then

(a) $v = \sqrt{5gl}$ (b) $v = \sqrt{7gl}$

(c) velocity of the ball at highest point is $\sqrt{gl}$ (d) velocity of the ball at the highest point is $\sqrt{3gl}$

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Exercise – II

ASSERTION & REASON , COMREHENSION & MATCHING TYPE

Assertion and Reason

In the following question, a statement of Assertion (A) is given which is followed by a corresponding statement of reason (R). Mark the correct answer out of the following options/codes.

(a) If both (A) and (R) are true and (R) is the correct explanation of (A).

(b) If both (A) and (R) are true but (R) is not correct explanation of (A).

(c) If (A) is true but (R) is false.

(d) If both (A) and (R) are false.

(e) If (A) is false but (R) is true.

1. A : No force is required to move a body in a complete circle.
   R : Work done by external force in a closed path is zero

2. A : A force does negative work when body is slowing down on which it acts.
   R : Negative work done by a force decreases the kinetic energy of the body.

3. A : If potential energy stored in a stretched spring is positive then the potential energy stored in the compressed spring is negative.
   R : Potential energy in the unstretched state is zero.

4. A : Work done by a force is same in all inertial frames.
   R : Work is a scalar quantity.

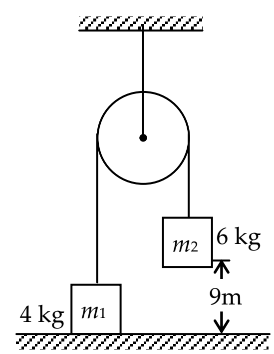
5. A : Net work done by static friction is always zero on the two bodies in contact.
   R : There is no relative motion between the bodies.

6. A : Mountain roads rarely go straight up.
   R : It requires large amount of work to be done.

Passage Based Questions

Passage I

Consider the arrangement shown in figure. An object of mass m_1 = 4 kg is connected to another object of mass m_2 = 6 kg by a massless string that passes over a massless and frictionless pulley. Initially, the 4 kg object is in contact with the floor and the 6 kg object is held at rest at a height 9m above the floor. At t = 0, the 6 kg object is released (Take g = 10 m/s^2)



1. Speed of the 4 kg object just when the 6 kg object hits the floor is

(a) 6 m/s (b) 8 m/s

(c) 4.5 m/s (d) 7.2 m/s

2. Tension in the string at t = 2.5s is

(a) 100 N (b) 18 N (c) 48 N (d) zero

3. Tension in the string at t = 3.2 s is

(a) 100 N (b) 24 N (c) 48 N (d) zero

4. Height of the 4 kg object from the floor when the 6 kg object is moving at 4 m/s is

(a) 5.2 m (b) 1.6 m (c) 2 m (d) 4 m

5. Maximum height above the floor to which 4 kg object rises is

(a) 8.2 m (b) 10.8 m (c) 9 m (d) 11.4 m

Passage II

In a diatomic molecule, potential energy of two atom is approximately given by

$$U(r) = \frac{A}{r^6} + \frac{B}{r^{12}}$$

where r is the interatomic spacing and A and B are positive constants. Assuming the interatomic force to be conservative, the corresponding force law is also called Lennard-Jones Law. It can be made out from the interatomic force law that atoms exert attractive force on each other as the separation between them reduces. The force law also tells that the force is repulsive at very short separations.

1. Equilibrium distance between the two atoms in a diatomic molecules can be expressed as

(a) $\frac{A}{B}$ (b) $\left(\frac{2B}{A}\right)^{1/6}$ (c) $\left(\frac{A}{B}\right)^{1/6}$ (c) $\left(\frac{A}{2B}\right)^{1/6}$

2. If the interatomic separation between the atoms of a diatomic molecule is equal to its equilibrium value, this equilibrium will be

(a) unstable

(b) neutral

(c) stable

(d) uncertain in its behaviour, *i.e.*, its stable, unstable or neutral nature cannot be predicted

3. If the interatomic separation is equal to its equilibrium value, potential energy will then have

(a) negative sign and maximum magnitude (b) positive sign and maximum magnitude

(c) positive sign and maximum magnitude (d) none of these

4. If the interatomic separation is equal to its equilibrium value, what minimum energy must be given to the molecule in order dissociate it?

(a) $\frac{B^2}{4A}$ (b) $\frac{A^2}{B}$ (c) $\frac{B^2}{2A}$ (d) $\frac{A^2}{4B}$

5. If the interatomic separation has such a value that potential energy is zero, interatomic force will then be

(a) attractive

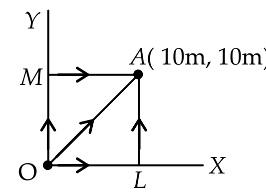
(b) repulsive

(c) zero

(d) uncertain regarding its sign, *i.e.*, its attractive or repulsive nature cannot be determined.

Passage III

Force acting on a particle moving in the *X*-*Y* plane is $\vec{F} = (y\hat{i} + x^2\hat{j})N$, *x* and *y* are in metre. Ass shown in figure (A), the particle moves from the origin '*O*' to point *A*. (10m, 10m). The figure shows three paths, *OLA*, *OMA* and *OA* for the motion of particle from *O* to *A*.



1. Which of the following correct?

(a) The given force is conservative

(b) The given force is non-conservative

(c) Conservative or non-conservative nature of force can be predicted on the basis of given information

(d) There is equal probability for the force being conservative or non-conservative

2. Along which of the three paths is the work done maximum?

(a) OMA

(b) OA

(c) OLA

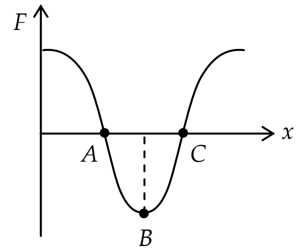
(d) Work done has the same value for all the three paths

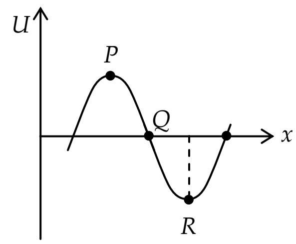
3. Work done for motion along the path *OA* is nearly

(a) 1000 J (b) 629 J (c) 383 J (d) 437 J

Matching Type Questions

1. F-x and corresponding U-x graph are shown in figure. Three points A, B and C in F-x graph may be corresponding to P, Q and R in the U-x graph. Match the following

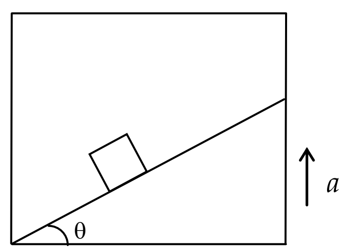




Column I	Column II
(A) *A*	(P) *P*
(B) *B*	(Q) *Q*
(C) *C*	(R) *R*
	(S) None of these

(a) A-P; B-Q; C-R (b) A-R; B-S; C-P (c) A-R; B-Q; C-P (d) A-P; B-S; C-R

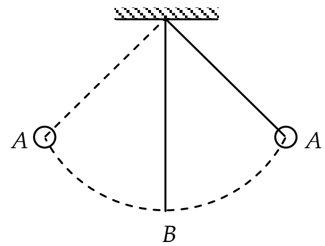
2. A block of mass m is stationary with respect to a rough wedge as shown in figure. Starting from rest in time t, (m = 1 kg, □ = 30°, a = 2 m/s^2, t = 4s) work done on block



Column I	Column II
(A) By gravity	(P) 144 J
(B) By normal reaction	(Q) 32 J
(C) By friction	(R) 160 J
(D) By all the forces	(S) 48 J
	(T) None of these

(a) A-R; B-P; C-S; D-Q (b) A-T; B-P; C-S; D-Q
(c) A-T; B-S; C-Q; D-S (d) A-R; B-S; C-Q; D-S

3. A pendulum is released from point *A* as shown in figure. At some instant net force on the bob is making an angle □ with the string. Then match the following



(A) For □ < 90°	(P) Particle may be moving along *BA*
(B) For □ > 90°	(Q) Particle may be moving along *CB*
(C) For □ = 90°	(R) Particle is at *A*
(D) For □ = 0°	(S) Particle is at *B*
	(T) None

(a) A-Q; B-Q; C-S; D-R

(b) A-P; B-Q; C-R; D-S

(c) A-P, Q; B-T; C-R; D-S

(d) A-P; B-Q; C-S; D-R

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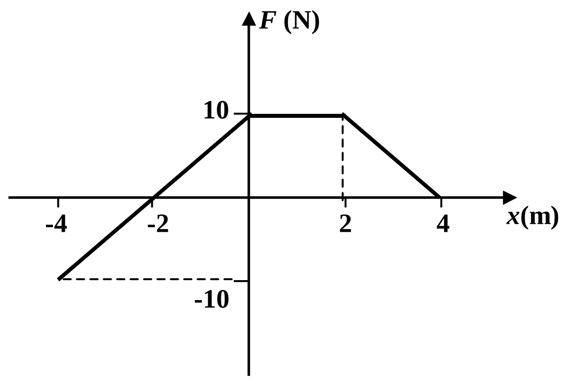
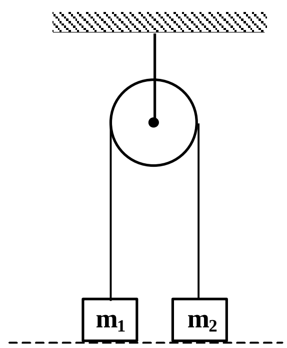
Exercise - III

SUBJECTIVE TYPE

1. A 200 g ball thrown vertically up with an initial speed of 20 m/s reaches a maximum height of 18 m. Find
   (a) the change in its kinetic energy
   (b) the work done by gravity
   (c) Are the two quantities just calculated equal? Explain why or why not?
2. A force varies the position as shown in the figure. Find the work done by it from
   (a) x = -4 to +4m (b) x = 0 to –2 m

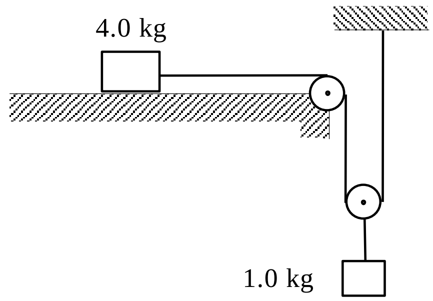


3. Two block of masses m_1 = 5 kg and m_2 = 2 kg hang on either side of a frictionless cylinder as shown in the figure. If the system starts at rest, what is the speed of m_1 after it has fallen 40 cm?

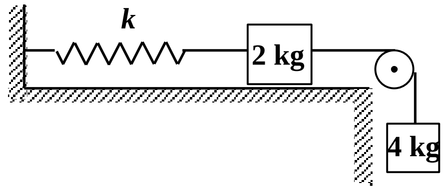


4. A projectile is fired at 25 m/s in a direction 60° above the horizontal from a rooftop of height 40 m. Use energy considerations to find
   (a) the speed with which it lands on the ground
   (b) the height at which its speed is 15 m/s

5. Consider the situation as shown in figure. The system is released from rest and the block of mass 1.0 kg is found to have a speed 0.3 m/s after it has descended through a distance of 1 m. Find the coefficient of kinetic friction between the block and the table.



6. A 2 kg block slides on a frictionless horizontal surface and is connected on one side to a spring (k = 40 N/m) as shown in the figure. The other side is connected to a 4 kg block that hangs vertically. The system starts from rest with the spring un-extended.

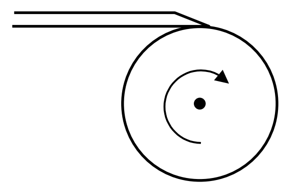


(a) What is the maximum extension of the spring?

(b) What is the speed of the 4 kg block when the extension is 50 cm?

7. A 75 kg parachutist is attached to an 8 kg parachute. She jumps from a plane flying at 140 km/h at an altitude of 1 km and immediately opens the parachute. If she lands vertically at 7 m/s, find the work done by the parachute on the air?

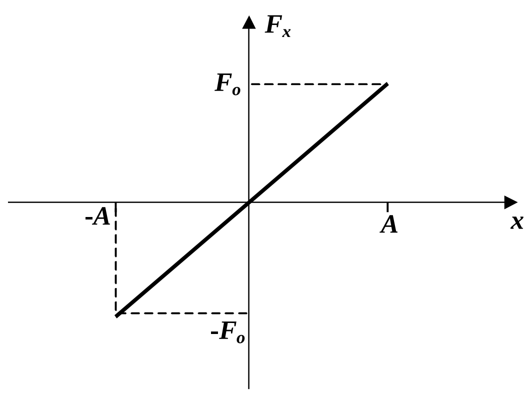
8. A tool being sharpened is held against a grinding wheel of radius 4 cm with a force of 20 N directed radially inward, as shown in the figure. If the coefficient of kinetic friction is 0.4, how much work is done by the wheel on the tool in 12 revolutions?



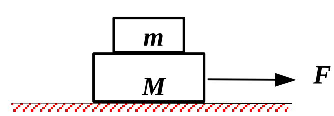
9. The variation of a force with position is depicted in figure. Find the work from

(a) $x = 0$ to $x = -A$

(b) $x = +A$ to $x = 0$.



10. A block of mass m is kept over another block of mass M and the system rests on a horizontal surface. A constant horizontal force F acting on the lower block produces an acceleration $\frac{F}{2(M+m)}$ in the system, the two blocks always move together.

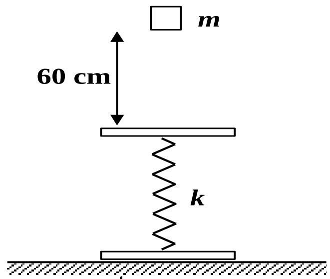


(a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface.

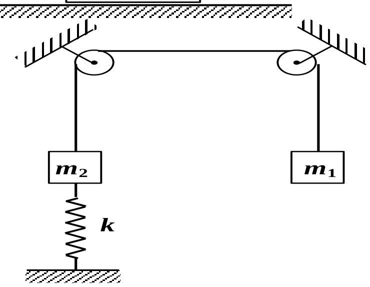
(b) Find the frictional force acting on the smaller block.

(c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement d of the system.

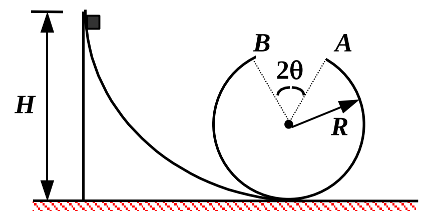
11. A 5 kg block is dropped from a height of 60 cm above the top of a vertical string whose stiffness constant is k = 1230 N/m. Find the maximum compression.



12. Two blocks with masses m_1 = 5kg and m_2 = 3 kg are connected by a light thread that passes over frictionless pulleys shown in figure. The smaller mass is attached to a spring (k = 32 N/m). If the system starts at rest with the spring unextended, find
(a) the maximum displacement of the larger mass
(b) the speed of the larger mass after it has fallen 1m.



13. A small object slides without friction from the height $H = 50$ cm and then loops the vertical loop of radius $R = 20$ cm from which a symmetrical section of angle 2θ has been removed. Find angle θ such that after losing contact at A and flying through the air, the object will reach point B.



14. A chain of length l and mass m lies on the surface of a smooth sphere of radius $R > l$ with one end tied to the top of the sphere.
(a) Find the gravitational potential energy of the chain with reference level at the centre of the sphere.
(b) Suppose the chain is released and slides down the sphere. Find the kinetic energy of the chain, when it has slide through an angle θ.
(c) Find the tangential acceleration $\frac{dv}{dt}$ of the chain when the chain starts sliding down.

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IIT – JEE PROBLEMS

A. Multiple Choice Questions with ONE correct answer

1. A particles of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particles by the force acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t (c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero

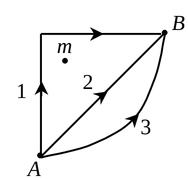
2. A wind–powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed V, the electrical power output will be proportional to

(a) V (b) V^2 (c) V^3 (d) V^4

3. An ideal spring with spring–constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially un-stretched. Then the maximum extension in the spring is

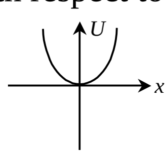
(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$ (c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

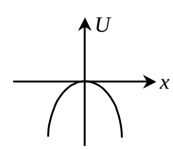
4. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths $1, 2$ and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3

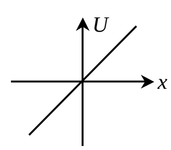


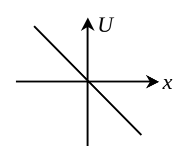
(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$ (c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

5. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x

(a) 

(b) 

(c) 

(d) 

B. Multiple Choice Questions with ONE or MORE THAN ONE correct answer

6. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

7. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. If follows that

(a) its velocity is constant (b) its acceleration is constant

(c) its kinetic energy is constant (d) it moves in a circular path

8. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its

(a) maximum potential energy is 100 J (b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J (d) maximum potential energy is zero

9. A force $FG = -K\left(y\hat{i} + x\hat{j}\right)$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive $x-$axis to the point $(a, 0)$, and then parallel to the $y-$axis to the point (a, a). The total work done by the force F on the particle is

(a) $2K\,a^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

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Answers

Exercise - I

Only One Option is correct

1.	(a)	2.	(b)	3.	(c)	4.	(d)	5.	(a)
6.	(d)	7.	(c)	8.	(d)	9.	(a)	10.	(d)
11.	(c)	12.	(c)	13.	(b)	14.	(c)	15.	(b)
16.	(d)	17.	(c)	18.	(d)	19.	(c)	20.	(c)
21.	(a)	22.	(a)	23.	(c)	24.	(a)	25.	(c)

More Than One Choice Correct

1.	(c, d)	2.	(a, b, d)	3.	(a, d)	4.	(a, c)	5.	(b, c, d)
6.	(b, d)	7.	(a, b, c, d)	8.	(a, c, d)	9.	(a, c, d)	10.	(b, d)

Exercise - II

Assertion and Reason

1.	(d)	2.	(e)	3.	(e)	4.	(e)	5.	(a)
6.	(c)	7.	(c)						

Passage I

1.	(a)	2.	(c)	3.	(d)	4.	(d)	5.	(b)

Passage II

1.	(b)	2.	(c)	3.	(a)	4.	(d)	5.	(b)

Passage III

1.	(b)	2.	(c)	3.	(c)

Matching Type Questions

1.	(c)	2.	(b)	3.	(c)

Exercise - III

Subjective Type

1. (a) – 40 J; (b) –36 J (c) No, air resistance
2. (a) 30 J; (b) -10 J
3. 1.83 m/s
4. (a) 37.5 m/s; (b) 60.4 m
5. 0.115
6. (a) 1.96 m; (b) 2.21 m/s
7. 8.74×10^5 J
8. 24.1 J
9. (a) $\frac{F_o A_o}{2}$ (b) $-\frac{F_o A_o}{2}$
10. (a) $\frac{F}{2(M+m)g}$; (b) $\frac{mF}{2(M+m)}$; (c) $\frac{mFd}{2(M+m)}$
11. 26.6 cm
12. (a) 1.23 m (b) 0.95 m/s
13. 60°.
14. (a) $\frac{mR^2g}{l}.\sin(\frac{l}{R})$;(b) $\frac{mR^2g}{l}.[\sin(\frac{l}{R}) + \sin\theta - \sin(\theta + \frac{l}{R})]$;(c) $\frac{Rg}{l}\left[1-\cos\left(\frac{l}{R}\right)\right]$

Exercise - IV

IIT-JEE Level Problem

Section - A

1.	(c)	2.	(c)	3.	(b)	4.	(b)	5.	(b)

Section - B

6.	(c)	7.	(c, d)	8.	(c, d)	9.	(c)

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